Mathematical Analysis
& Convex Optimization

Vol. xxxxxx
https:\\ maco.lu.ac.ir
DOI: DOI: 10.22034/maco.6.1.5

Research Paper

HYBRID MULTI-POPULATION GENETIC ALGORITHM FOR MULTI-CRITERIA PORTFOLIO SELECTION

MOHAMMAD MIRABI AND HOSSEIN GHANEAI*

ABSTRACT. One of the most important decision for each investor is how to select the best financial assets to have more profit in future. Portfolio selection is a knowledge-based decision-making process that guide investor to select an optimal or near optimal basket of investment. During recent decades so many approaches are developed in this context. Heuristics and meta-heuristics have the major part of papers that are considered by researchers. Local search, ant colony, simulated annealing and neural network are some of mentioned methods in the literature. In this research, we develop one meta-heuristic method based on genetic and simulates annealing to select portfolio for risky investor with multi criterial approach. The approach that is shown begins with a multipopulation of created solutions and works its way up to the ultimate solution. Studies comparing our approach to other newly developed methods in the literature show that it can locate a strong portfolio with a high rate of return.

Keywords: Portfolio Selection, Genetic Algorithm, Multi Criterial, Genetic Operators, Meta-Heuristic.

1. Introduction and Background

Portfolio selection is one of the most concern of any investor and the goal is how to asset allocation in various sections to have efficient investment portfolio. A variety of riskier investments can be made in gold and silver, financial institutions, bonds, the stock market, the housing market, and the foreign exchange market. Decision-making scenarios can be fully definite, highly dangerous, or entirely unknown, and heuristics or optimization can be used as problem-solving strategies. Heuristics and meta-heuristics are mainly attending in the recent researches because of complexity of portfolio problem based on computational theory.

In computational theory, problems may be divided into two primary classes: P and NP. A deterministic algorithm can solve a problem of complexity class P in polynomial time. Then, the solution is not too difficult. P class includes minimum spanning tree, shortest path, maximum flow network, maximum bipartite matching, and continuous linear programming models. A nondeterministic method can solve an NP difficulty class problem in polynomial time.

If every other decision issue in class NP can be solved in polynomial time, then the problem is considered NP complete. Optimization issues that have NP-complete decision problems attached to them are known as NP-hard problems. NP-hardness characterizes most academic and practical optimization issues as well as many real-world challenges. Portfolio selection

Date: Received: , Accepted: .

*Corresponding author.

is one of the NP hard issues, along with others like routing and covering, sequencing and scheduling, packing/cutting and knapsack, assignment and placement, and so on [7].

Since there are no provably efficient algorithms for NP-hard problems, meta-heuristics with pure or hybrid structures have a wide range of uses in problem solving.

Learning strategies are used to structure information in order to find efficiently near-optimal solutions. A meta-heuristic is defined as an iterative generation process that guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space

During last decades, many heuristics methods have been developed for portfolio selection [5]. In this paper we introduce a population-based method called multi population genetic algorithm to solve portfolio selection. Presented genetic is hybridized with simulated annealing to enrich the capability of algorithm, therefore we called it as hybrid multi population genetic algorithm (HMPGA). The rest of paper is organized as follows: Section 2 introduces literature review of portfolio selection and some efficient approaches developed to solve it. The HMPGA is defined in section 3. Section 4 devotes to verification and comparison study and finally section 5 concludes the research.

2. **Literature Review**

Portfolio selection is a branch of modern portfolio theory (MPT) that deal with selecting a portfolio of assets such that the extracted criteria is optimized within reasonable level of risk. Choice of criteria depends on the investor but this field of study is mainly multi-criteria decision making (MCDM) considering financial and non-financial issues [12, 11].

Pioneer work in portfolio selection (mainly security selection) is due to [16]. He assumed that for each investor expected return is desirable and variance of return is undesirable therefore his work is famous as mean-variance analysis. Based on his research the variance from expected return is assumed as the risk of portfolio [8].

In general portfolio selection aim to allocation wealth to a set of assets according to the investor viewpoint [14, 15]. The research questions are as follows:

(1) What are the main criteria and constrains to select portfolio?
(2) Which approach must be used to solve the problem?

Wide range of criteria are considered in portfolio selection bur expected return (EV) is commonly used [23]. Ogryczak [22], developed an MCDM model with risk consideration and Abdelaziz et al. [1] considered multi-objective stochastic programming model for portfolio selection. Extracting potential criteria and sub-criteria is one approach considered by Mihail et al. [17] and Jeng and Huang [8, 9] represented a systematic MCDM with modified Delphi method for this problem. By a fast glance in the literature, it can be seen that MCDM approach has a main part of research in portfolio field [10, 26].

Portfolio selection categorized as Np-Hard problem therefore meta-heuristic approaches are very applicable to solve them. Doerner et al. [6] introduced Pareto Ant Colony Optimization as an effective approch to solve the portfolio selection problem and compares its performance to other heuristic approaches. Some algorithms have been developed with genetic algorithm [19, 20, 21]. Chen and Zhu [4] developed a particle swarm optimization (PSO) for financial investment. They compared PSO with genetic algorithm and showed its capability to find near optimal solution. Liu and Yin [3] proposed A portfolio forecasting model based on particle swarm optimization(PSO) algorithm with automatic factor scaling. Kumar and Mishra [13] proposed a novel co-variance guided artificial bee colony algorithm for portfolio optimization.

Xidonas et.al [25] proposed an integrated decision support system in python programming language to find an optimal portfolio. The proposed DSS consists of two phases: security selection phase and portfolio optimization phase. They tested their approach in several stock markets and various sectors to show the capability of it. Finally Salehpoor and Zavardehi [24] developed a new decision making method of portfolio optimization(PO) issues in different risk measures by using new evolutionary computing method and cardinality constrains which was mentioned as hybrid meta-heuristic algorithms.

According to Blum and Andrea [2], meta-heuristics possess the following fundamental characteristics:
The objective is to effectively explore the search space in order to identify close to optimum solutions. Meta-heuristics are tactics that "guide" the search process.

- Meta-heuristic algorithms encompass a variety of techniques, ranging from basic local search criteria to intricate learning procedures.
- Meta-heuristic algorithms are often non-deterministic and approximate.
- They could include safeguards against being stuck in restricted regions of the search space.
- Meta-heuristics' fundamental ideas enable an abstract level description.
- Meta-heuristics don't focus on a particular issue.
- Domain-specific knowledge may be utilized by meta-heuristics in the form of heuristics that are governed by the higher level approach.
- More sophisticated meta-heuristics of today employ search experience—embodied in a memory—to direct the search.

There are two primary types of meta-heuristics:

(1) Trajectory techniques such as variable neighborhood search, tabu search, simulated annealing, and basic local search.
(2) Population-based techniques such as particle swarm optimization, ant colony optimization, and genetic algorithms. Based on above properties, we are going to develop one population-based hybrid meta-heuristic to overcome portfolio problem.

3. Mathematical Model

To model our problem, we define the following parameters:
or_{it}: optimistic return at the end of time period t per dollar invested in sector i.
mr_{it}: most likely return at the end of time period t per dollar invested in sector i.
pr_{it}: pessimistic return at the end of time period t per dollar invested in sector i .
l_{it}: the probability of liquidity power at the end of time period t in sector i.
σ_{ij}: covariance between r_i and r_j.
N_{max}, N_{min}: maximum and minimum number of sectors that must be invested respectively (diversity constraint).
V_{max}: maximum acceptable risk (based on risk taking of an investor).
R_{min}: minimum expectable return (based on investor and also inflation rate).
n: number of sectors.
T: number of time periods.
x_i: decision variable repre.sents relative amount invested in sector i.
We define:

$$r_{it} = \frac{or_{it} + 4 \times mr_{it} + pr_{it}}{6}. \tag{3.1}$$

Now we present mathematical model as follow:

$$\max R = \sum_{i=1}^{n}\sum_{t=1}^{T} r_{it} \times x_i, \tag{3.2}$$

$$\max LP = \sum_{i=1}^{n}\sum_{t=1}^{T} l_{it} \times x_i, \tag{3.3}$$

$$\min V = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} \times x_i \times x_j. \tag{3.4}$$

Subject to:

$$\sum_{i=1}^{n} x_i = 1 \quad for\ t = 1,\ 2,\ \ldots,, \tag{3.5}$$

$$R \geq R_{min}, \tag{3.6}$$

$$V \leq V_{max}, \tag{3.7}$$

$$N_{min} \leq \sum_{i=1}^{n} I_i \leq N_{max}, \tag{3.8}$$

$$I_i = \begin{cases} 1 & if\ x_i > 0 \\ 0 & if\ x_i = 0 \end{cases},\ i = 1, 2, \ldots, n, \tag{3.9}$$

$$0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, n. \tag{3.10}$$

Equation (3.1) calculates expected average of return in sector i. Equations numbers (3.2), (3.3) and (3.4) maximum overall return and liquidity power and minimum overall variance as a risk indicator. If an investor does not care about the risk and just think about return the equation (3.4) can be eliminated (for risk takers). Equations (3.5) to (3.10) is about assumed constrains and all are clear.

4. Hybrid Multi Population Genetic Algorithm

Based on the life of a lion, a meta-heuristic is presented. Lions reside in prides, as opposed to the colonies of other felids. A pride is made up of one male leader, three or more males, related females, and their pups. The amount of food and water available determines pride size. Lower pride is the outcome of fewer resources. The pride's strongest lion, Commander, is in charge of defending both their territory and the cubs. He assumes this position until he is defeated and his pride is taken by another powerful man. The group's main hunters are female lions.

Around the age of two, male cubs are required to leave the pride. Until they have the strength to confront male lions from other prides, they join small groups.

This type of treatment encourages the development of a method for solving portfolio issues known as hybridized population genetics (MPGA) with simulated annealing (HMPGA). HMPGA is a population-based algorithm as it first requires a starting population of solutions. It is important to remember that every solution needs to work around every restriction.

To generate each member of initial population, the following steps are considered:

(1) Set $x_z = 0$ for $z = 1, 2, \ldots, n$
(2) Used random integer generator to earn k (integer between N_{min} and N_{max})

(3) For $i = 1 : k - 1$
Used random integer generator to earn j (integer between 1 and n)
If $x_j > 0$ replace j with the nearest j (between 1 and n) that $x_j = 0$
Endif
$x_j =$ Random variable between $(0, 1)$
If $\sum_{z=1}^{n} x_z \geq 1$
$x_j =$ Random variable between 0 and 1
End if
End for
(4) Used random integer generator to earn j (integer between 1 and n)
If $x_j > 0$ replace j with the nearest j (between 1 and n) that $x_j = 0$
Endif
(5) $x_j = 1 - \sum_{z=1}^{n} x_z$
(6) Extract all x_z $z = 1, 2, \ldots, n$

Subsequently, every answer that was obtained is sorted into groups (prides). Every group's solutions must be more than two. Based on the weighted arithmetic mean of the three goal functions, the optimal solution for each group is the one with the highest fitness function, as follows:

$$Fitness\ Function_k\ (FF_k) = w_1 \times R_k + w_2 \times LP_k - w_3 \times V_k. \tag{4.1}$$

w_j $(= 1, 2$ and $3)$ is calculated by the investor viewpoint. The commander, who is the best FF in each group, is the solution. In the next stages, heuristic mutation operations or order crossover operations are used in each group to create new solutions, or offspring.

We employ crossover procedures with heuristic mutation, similar to those already out by Mirabi [18]. These operations are described in Figures 1 and 2. It is important to keep in mind that the order crossover operator is limited to solutions of the same size, meaning the same number of sections chosen for investment.

		Selected substring			
Parent 1	0.15	0.1	0.3	0.2	0.25
Parent 2	0.25	0.3	0.15	0.15	0.15
Offspring 1	0.15	0.3	0.15	0.15	0.25

Figure 1. The order crossover operator.

Each new solution (FF_{new}) in each group that pass constraints, challenges all commanders and if defeats one of them (is better than it), becomes the new commander and replaced with the worst member of related group (FF_{worst}). But if is not better that at least one commander, but is better than the worst solution of group that is born, replaced it with the probability of $e^{-\frac{1}{FF_{new} - FF_{worst}}}$. The replacement helps us to find new rout and rise our chance to find a better solution (based on simulated annealing approach).

Let us use the following notations for our algorithm:
m: number of all initial solutions (all population of lions). m must be more than 4 because we need at least two groups with size of at least 2.

Select three genes randomly

Parent	0.1	0.12	0.35	0.18	0.25
Offspring 1	0.1	0.12	0.35	0.18	0.25
Offspring 2	0.35	0.12	0.1	0.18	0.25
Offspring 3	0.1	0.12	0.25	0.18	0.35
Offspring 4	0.25	0.12	0.35	0.18	0.1
Offspring 5	0.35	0.12	0.25	0.18	0.4
Offspring 1	0.25	0.12	0.1	0.18	0.35

FIGURE 2. The heuristic mutation operator.

n: number of groups (prides). Each group has at least two solutions and therefore $2 \leq n \leq [m/2]$.
k: number of iterations that we need to generate new solutions in each group that challenge other commanders to replacement.

The pseudo code of HMPGA is as follows:

Input m, n and k
Describe fitness function (combination of e presented objective functions)
Generated m initial solutions by random (should pass constraints)
Generate n pride numbered 1 to n
For $j = 1 : [m/n] + 1$
    For $i = 1 : n$
        If there are unallocated solution,
            Select one solution by random and allocate it to pride numbered i
        End if
    End for
End for
For $i = 1 : n$
    Select the best solution in pride i based on the fitness function and called it as commander (commander = arg max (fitness))
End for
For $n = 1 : k$
    For $j = 1 : n$
        Using a heuristic mutation of the commander or an order crossover between the commander and one pride member, generate some new solutions at random in pride j.
        Compare the new solution with all commanders using FF if it meets all requirements. Call it the new commander and replace it with the least desirable solution in the pride if the new solution outperforms one of them (based on fitness function).
        Otherwise if it is better than worst solution of its group, replace it with the probability of $e^{-\frac{1}{FF_{new}-FF_{worst}}}$
    End for
End for

Compare all commanders and select the best for the final solution based on FF.

5. Comparison Study

In this section, a computational study is carried out to compare the HMPGA with the best developed heuristics in portfolio area. Extracted meta-heuristics must be adoptable to MCDM and constrained portfolio. Two heuristics as follows were selected for comparison:

Bee colony algorithm (BCA) developed by Kumar and Mishra (2017)

Electromagnetism-like algorithm (EM) developed by Salehpoor and Zavardehi (2019)

We select 100 factories on the Iranian stock exchange. Each factory assumed as a sector for investment. We set N_{max} to 50, 70, 80 and 100 and N_{min} to 5, 10, 20 and 30 and also assume T=1. Sector's return considered as fitness function and for evaluation study we use PM index defined as follow:

$$PM = \frac{R_{best} - R_i}{R_{best}} \tag{5.1}$$

where the return obtained by a given algorithm is R_i and R_{best} is the return of the best solution obtained by all algorithms. In MATLAB, the programs are coded. Due to the stochastic nature of algorithms, a standard method for experimentally comparing them involves performing many runs on the same task. All algorithms are performed ten times independently for equal conditions across the three approaches, with a stopping criterion based on a limited number of iterations (100000). After taking into account all tested setups, we end up with 150 issue cases and 16 different classes of problems.

In Table 1 Min, Max and the average PM of each method is shown. Also, the average time to solve 10 instances are given for each method. The columns labelled "Min" show, in subscript, the number of instances for which the algorithm solution was equal to the corresponding Best solution. As show in Table 1, HMPGA out performs others based on PM value and HMPGA and BCA almost have the same speed.

			HMPGA				**BCA**				**EM**			
Class of	N_{min}	N_{max}	Min	Average		Max	Min	Average		Max	Min	Average		Max
problem			PM	PM	Time	PM	PM	PM	Time	PM	PM	PM	Time	PM
1	5	50	0_4	0.06	3.11	0.12	0_4	0.03	3.43	0.01	0_2	0.11	4.45	0.15
2	5	70	0_5	0.05	2.48	0.10	0_4	0.06	3.54	0.13	0_2	0.09	4.28	0.12
3	5	80	0_5	0.04	2.39	0.09	0_5	0.05	2.58	0.01	0.01	0.12	3.53	0.17
4	5	100	0_6	0.02	2.11	0.09	0_3	0.05	2.25	0.05	0_2	0.07	3.28	0.08
5	10	50	0_7	0.01	3.55	0.00	0_2	0.05	4.01	0.02	0_1	0.03	4.44	0.00
6	10	70	0_9	0.02	2.58	0.10	0_3	0.06	3.14	0.14	0_2	0.08	4.23	0.10
7	10	80	0_9	0.02	2.50	0.12	0_4	0.06	2.55	0.06	0.00	0.07	3.56	0.07
8	10	100	0_6	0.04	2.45	0.05	0_5	0.07	2.50	0.14	0_1	0.18	3.31	0.14
9	20	50	0_6	0.06	3.57	0.13	0_3	0.09	3.50	0.13	0_1	0.14	5.43	0.16
10	20	70	0_9	0.03	3.22	0.14	0_3	0.08	3.59	0.03	0_1	0.11	4.58	0.15
11	20	80	0_9	0.01	2.48	0.01	0_2	0.05	2.34	0.02	0.01	0.16	4.25	0.03
12	20	100	0_8	0.01	2.59	0.01	0_5	0.06	2.10	0.10	0.01	0.19	4.53	0.18
13	30	50	0_5	0.04	4.21	0.09	0_4	0.08	4.24	0.11	0_2	0.12	5.40	0.18
14	30	70	0_6	0.04	3.57	0.11	0_4	0.04	3.29	0.13	0.01	0.07	5.55	0.17
15	30	80	0_6	0.03	3.37	0.00	0_4	0.08	3.59	0.15	0.00	0.09	5.11	0.18
16	30	100	0_7	0.02	3.41	0.08	0_3	0.05	3.38	0.01	0.01	0.11	4.48	0.19

TABLE 1. PM values for comparison studies between all algorithms (times are in minute).

Table 1 demonstrates that HMPGA and BCA are more competitor. For detail comparison between HMPGA and BCA, we should check that the differences between solutions of two algorithm are statistically significant or not. For this, the hypothesis that the population corresponding to the differences has mean (μ) zero can be tested; specifically, test the (null) hypothesis $\mu = 0$ against the alternative $\mu > 0$. This test is performed between two best methods based on Table 1 (HMPGA and BCA). It is assumed that the differences between solutions (fitness function or FF) is a Normal variable, and choose the significance level $\alpha = 0.05$. If the hypothesis is true, the random variable $T = (\overline{X}_1 - \overline{X}_2)/\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}$ has a t distribution with$\upsilon = (S_1^2/n_1 + S_2^2/n_2)^2/(\frac{(S_1^2/n_1)^2}{n_1-1} + \frac{(S_2^2/n_2)^2}{n_2-1})$ degrees of freedom. The critical value of c is obtained from the relation Prob($T > c$)= $\alpha = 0.05$. Table 2 shown this study. For more explanation, consider the first row of Table 2, corresponds to the sample size=$n_1 = n_2 = 10$, $\mu_0 = 0$, sample mean for HMPGA and BCA are 62.80 and 61.30 respectively. Sample standard deviation for HMPGA and BCA are 2.48 and 2.39 respectively. Since $t = 1.73 > T = 1.38$, we conclude that the difference is not statistically significant.

Class of			**Ave. FF or ($\overline{FF}$)**		**Ave. SD or (S)**					
problem	N_{min}	N_{max}	HMPGA	BCA	HMPGA	BCA	T	ν	t	sig.
1	10	5	62.80	61.30	2.48	2.39	1.38	18	1.73	No
2	10	10	50.60	57.05	2.42	2.96	-5.33	17	1.74	Yes
3	10	15	73.50	59.62	2.25	2.30	13.64	18	1.73	Yes
4	10	20	76.42	60.24	2.82	3.24	11.91	18	1.73	Yes
5	20	5	71.31	50.10	2.97	3.26	15.21	18	1.73	Yes
6	20	10	80.05	59.21	2.68	2.85	16.85	18	1.73	Yes
7	20	15	73.15	59.49	4.01	4.99	6.75	17	1.74	Yes
8	20	20	57.23	69.28	4.45	4.19	-6.24	18	1.73	Yes
9	30	5	62.67	68.23	4.27	4.42	-2.86	18	1.73	Yes
10	30	10	81.12	60.71	3.92	4.67	10.59	17	1.74	Yes
11	30	15	79.12	67.15	4.78	4.26	5.91	18	1.73	Yes
12	30	20	71.89	61.46	3.52	4.25	5.98	17	1.74	Yes
13	40	5	59.18	70.15	5.18	5.11	-4.77	18	1.73	Yes
14	40	10	78.83	59.19	5.51	6.07	7.58	18	1.73	Yes
15	40	15	70.02	60.40	6.02	6.90	3.32	18	1.73	Yes
16	40	20	69.63	79.15	4.97	5.59	-4.02	18	1.73	Yes
Ave: Average, SD: Standard deviation, Sig: Significant										
Each class contains 10 independent instances										

TABLE 2. Detail comparison between NGA and HGA (FF in percent).

Table 3 demonstrated that HMPGA outperformed BCA in 69% of all classes and all of differences are statistically significant except one. Also, BCA outperformed HMPGA in 31% of all classes that in all cases, differences are statistically significant.

To do a deep comparison between HMPGA and BCA Tukey honestly significance difference test can be used. It is a strong statistical tool to check significance by computing confidence interval similarly to the confidence interval for the difference of two means, but using the q distribution which avoids the problem of inflating :

$$\hat{x}_i - \hat{x}_j \pm q\,(,r,df_w)\sqrt{\frac{FF_w}{2} \times \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Table 3 summarized the outputs of this test.

		Critical q	**95% Conf Interval for $\mu_{HMPGA} - \mu_{BCA}$**		**Significant at 0.05**
	$\widehat{FF}_{HMPGA} - \widehat{FF}_{BCA}$	$\mathbf{q}(\alpha, r, df_w)$	Min	Max	
$HMPGA - BCA$	12.83	1.90	-9.06	9.11	Yes

Table 3. Tukey test results for HMPGA and BCA.

Table 3 demonstrated that HMPGA completely outperforms BCA.

6. Conclusion

We presented a population-based meta-heuristic approach to the portfolio selection issue in this study. We have named the approach we have shown the Hybrid Multi-Population Genetic Algorithm (HMPGA) since it is based on both the structure of simulated annealing and the lifestyle of lions. The best answer within each group is referred to as the commander, and the initial population of solutions is divided across several groups (prides). Every child in every group (acquired through mutation or crossover) challenges every commander to replace themselves with the poorest group member and take on the role of new leader. The optimal commander is the result of a finite number of repetitions. We pulled out two potent techniques—the electromagnetic-like algorithm and the bee colony algorithm—from the literature for the verification test. Based on the comparison study, HMPGA works very competitive to solve multi-criteria portfolio selection problems.

References

[1] F.B. Abdelaziz, B. Aouni and R. El Fayedh, Multi-objective stochastic programming for portfolio selection. *Eur. J. Oper. Res.*, **177**:1811–1823, 2007.

[2] C. Blum and R. Andrea, Metaheuristics in Combinatorial Optimization: Overview and Conceptual Comparison. *ACM Computing Surveys*, **35**:268–308, 2003.

[3] C. Liu and Y. Yin, Particle swarm optimised analysis of investment decision Cognit. *Syst. Res.*, **52**:685–690, 2018.

[4] Y. Chen and H. Zhu, PSO Heuristics Algorithm for Portfolio Optimization. *International Conference in Swarm Intelligence*, 2010.

[5] J. Doering, R. Kizys, A. Juan, A. Fito and O. Polat, Metaheuristics for rich portfolio optimization and risk management: Current state and future trend. *Operation research perspectives*, **6**, 2019. https://doi.org/10.1016/j.orp.2019.100121.

[6] K. Doerner, W. Gutjahr, R. Hartl, H. Strauss and C. Stummer, Pareto Ant Colony Optimization: A Metaheuristic Approach to Multi-objective Portfolio Selection. *Annals of Operations Research*, **131**:79–99, 2004.

[7] El-Ghazali, Talbi, *Meta-heuristics, from design to implementation.*, by John Wiley & Sons, Inc., Hoboken, New Jersey, 2009.

[8] X. Huang, Fuzzy chance-constrained portfolio selection. *Appl. Math. Comput.*, **177**:500–507, 2006.

[9] D.J.F. Jeng and K.H. Huang, Strategic project portfolio selection for national research institutes. *J. Bus. Res.*, **68**:2305–2311, 2015.

[10] G. Kou, X. Chao, Y. Peng, F.E. Alsaadi and E. Herrera-Viedma, Machine learning methods for systemic risk analysis in financial sectors. *Technol. Econ. Dev. Econ.*, **25**:1–27, 2019.

[11] G. Kou, D. Ergu, C. Lin and Y. Chen, Pairwise comparison matrix in multiple criteria decision making. *Technol. Econ. Dev. Econ.*, **22**:738–765, 2016.

[12] G. Kou, Y. Lu, Y. Peng and Y. Shi, Evaluation of classification algorithms using MCDM and rank correlation. *Int. J. Inf. Technol. Decis. Mak.*, **11**:197–225, 2012.

[13] D. Kumar and K. Mishra, Portfolio optimization using novel co-variance guided Artificial Bee Colony algorithm. *Swarm. Evol. Comput.*, **33**(1):119-130, 2017.

[14] B. Li, S.C. Hoi, D. Sahoo and Z.Y. Liu, On some non-linear elliptic differential-functional equations. *Acta Mathematica*, **1222**(a):104–123, 2015.
[15] T. Li, W. Zhang and W. Xu, A fuzzy portfolio selection model with background risk. *Appl. Math. Comput.*, **256**(b):505–513, 2015.
[16] H. Markowitz, portfolio selection. *the journal of finance*, **7**:77–91, 1952.
[17] D. Mihai, H. Alina, G. Grigorios, Criteria determining optimal portfolio selection: a case study. *Am. J. Econ. Bus. Adm.*, **5**:120-128, 2013.
[18] M. Mirabi, A novel hybrid genetic algorithm to solve the sequence dependent permutation flow-shop scheduling problem. *International Journal of Advanced Manufacturing Technology*, **71**:429-437, 2014.
[19] M. Mirabi and H. Ghaneai, Hybrid Multi-population Genetic Algorithm for Multi Criteria Project Selection. *Journal of Mahani Mathematical Research*, **11**(2):61–74, 2022.
[20] M. Mirabi, H. Ghaneai, S. mousavi and H. Tavakoli, Bitcoin price forecasting using hybrid genetic algorithm. *Mathematics and Computational Sciences*, **5**(2):34–48, 2024.
[21] H.B. Meybody, Sh.Mohammadi Ardakani and H. Ghaneai, A New Decision-Making Method Based on Shannon Entropy. *Iranian Journal of Operations Research*, **17**(1):1–19, 2026.
[22] W. Ogryczak, Multiple criteria linear programming model for portfolio selection. *Acta Mathematica*, **97**(1–4):143–162, 2006.
[23] M. Rahimnezhad, F. Rafiei and M. Ghezelbash, Portfolio selection: a fuzzy-ANP approach. *Financial Innovation*, **6**:2–34, 2020.
[24] B. Salehpoor and M. Zavardehi, A constrained portfolio selection model at considering risk-adjusted measure by using hybrid meta-heuristic algorithms. *Applied Soft Computing*, **75**:233–253, 2019.
[25] P. Xidonas, H. Doukas and E. Sarmas, A PYTHON-BASED MULTICRITERIA PORTFOLIO SELECTION DSS. *RAIRO-Oper. Res.*, **55**:S3009–S3034, 2021.
[26] H. Zhang, G. Kou and Y. Peng, Soft consensus cost models for group decision making and economic interpretations. *Eur. J. Oper. Res.*, **277**:964–980, 2019.

(Mohammad Mirabi) Department of Industrial Engineering, Meybod University, Meybod, Iran.

(Hossein Ghaneai) Department of Computer Engineering, Meybod University, Meybod, Iran.
Email address: h.ghaneai@meybod.ac.ir