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Research Paper

FUZZY STOCHASTIC DIFFERENTIAL SYSTEM

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ABSTRACT. In this paper, we study fuzzy stochastic differential equation initial value problems (IVPs). We obtain the existence and uniqueness theorem for a solution of the fuzzy stochastic differential equation (FSDE) under the Lipschitz condition. We present characterization theorems for the solution of a FSDE under the m.s. derivative-based interpretation, by the solution of a system of ODEs. Numerical examples are provided which connect the new results with previous findings.

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1. Introduction

Stochastic modeling, control, and optimization have played a crucial role in many applications. Since systems in the real world often need to run for a long period of time, an important problem concerns stability of such systems. In modeling, analyzing, and predicting behaviors of physical and natural phenomena, greater and greater emphasis has been placed upon fuzzy stochastic methods. This is due to combinations of complexity, two kinds of uncertainty-randomness and fuzziness, and ignorance which are present in the formulation of a great number of these problems. A large class of physically important problems is described by fuzzy stochastic differential systems.

2. Preliminaries

Definition 2.1. Let $K_F(R^n)$ denote the family of all non-empty, compact, convex subsets of R^n. Denote by E^n the set of $\widetilde{u} : R^n \to [0,1]$ such that $\widetilde{u}$ satisfies $(i) - (iv)$ mentioned next:

(i) $\widetilde{u}$ is normal that is, there exists an $y_0 \in R^n$ such that $\widetilde{u}(y_0) = 1$,
(ii) $\widetilde{u}$ is fuzzy convex,
(iii) $\widetilde{u}$ is upper semi continuous,
(iv) $[\widetilde{u}]^0 = \overline{\{y \in R^n : \widetilde{u}(y) > 0\}}$ is compact .

We denote the $\alpha-$ level set $[\widetilde{u}]^\alpha = \{y \in R^n : \widetilde{u}(y) \geq \alpha\}$ for $0 < \alpha \leq 1$. Clearly the α-level sets $[\widetilde{u}]^\alpha \in K_F(R^n)$.

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Definition 2.2. [4]. Let I be a real interval. A mapping $\widetilde{y} : I \longrightarrow E^n$ is called a fuzzy process and its $\alpha-$ level set is denoted by

$$[\widetilde{y}]^\alpha = [\underline{y}^\alpha, \overline{y}^\alpha] \quad t \in I, \quad 0 < \alpha \leq 1.$$

Let $\widetilde{x},\ \widetilde{y} \in E^n$.

Definition 2.3. [1]. If $A \in K_F(R^n)$, the family of all nonempty compact convex subsets of R^n, I_A is its characteristic function, then $I_A \in E^n$. A linear structure in E^n is defined as usual. Let $u, v \in E^n$, and set

$$D(u,v) = \sup_{0 \leq \alpha \leq 1} d([u]^r, [v]^r),$$

where $[u]^\alpha = \{x \in R^n \mid u(x) \geq \alpha\}, 0 < \alpha \leq 1$, is the α-level set of u.

Definition 2.4. [1]. d is the Hausdorff metric defined in $K_F(R^n)$, i.e.

$$d(A,B) = \max(\sup_{a \in A} \inf_{b \in B} \mid a - b \mid, \sup_{b \in B} \inf_{a \in A} \mid a - b \mid),$$

for all $A, B \in K_F(R^n)$, where $\mid . \mid$ denotes the usual Euclidean norm in R^n.

Definition 2.5. [1]. The norm $\parallel u \parallel$ of a fuzzy number $u \in E^n$ is defined by

$$\parallel u \parallel = D(u, \hat{0}) = \parallel [u]^0 \parallel = \sup_{a \in [u]^0} \mid a \mid,$$

where $\hat{0}$ is the fuzzy number in E^n which membership function equals 1 at 0 and zero elsewhere.

(E^n, D) is a complete metric space [4].

Let (Ω, A, P) be a complete probability space. A fuzzy random variable (f.r.v. for short) is a Borel measurable function $X : (\Omega, A) \to (E^n, D)$. If $E \parallel X \parallel < \infty$, then the expected value EX exists.

Let $L_2 = \{X \mid X \text{ is an f.r.v. with } E \parallel X \parallel^2 < \infty\}$.

Definition 2.6. [1]. Two f.r.v.'s X and Y are called equivalent if $P(X \neq Y) = 0$. The all equivalent elements in L_2 are identified.

Definition 2.7. [1]. Define

$$\rho(X,Y) = (ED^2(X,Y))^{1/2}, X, Y \in L^2.$$

The norm $\parallel X \parallel^2$ of an element $X \in L^2$ is defined by

$$\parallel X \parallel_2 = \rho(X, \hat{0}) = (E \parallel X \parallel^2)^{1/2}.$$

(L_2, ρ) is a complete metric space [2],

Corollary 2.2 and ρ satisfies that

$$\rho(X+Z, Y+Z) = \rho(X,Y), \rho(\lambda X, \lambda Y) = \mid \lambda \mid \rho(X,Y), \tag{2.1}$$

$$\rho(\lambda X, kX) \leq \mid \lambda - k \mid \parallel X \parallel_2, \tag{2.2}$$

for any $X, Y, Z \in L_2$ and $\lambda, k \in R$.

Definition 2.8. [1]. Let $u, v \in E^n$. If there exists a $w \in E^n$ such that $u = v + w$ then we call w the H-difference of u and v, denoted by $u \ominus v$.

Because $u \ominus v : E^n \times E^n \to E^n$ is continuous, if X and Y are f.r.v.'s and the H-difference of X and Y exists a.s., i.e. $P(X \ominus Y \text{ exists}) = 1$,

then $X \ominus Y$ is an f.r.v. and $X \ominus Y \in L_2$ provided $X, Y \in L_2$. Let $(X_n)_{n>1}$ be a sequence in L_2.

Definition 2.9. [1]. We call that X^n converges in mean square or m.s.converges to X as $n \to 1$ if $\rho(X_n, X) \to 0$, write $X_n \vec{m.s} X$ or $\lim_{n\to\infty} X_n = X$.

Definition 2.10. [1]. Let T be a finite or an infinite interval in R. A mapping $X : T \to L_2$ is called a second-order fuzzy stochastic process (f.s.p. for short). If X is continuous at a $t \in T$ with respect to the metric ρ then we call X continuous in mean square or m.s. continuous at t. If X is m.s. continuous at every $t \in T$ then we call X m.s. continuous. For above further details the reader is refered to [1, 2].

Definition 2.11. [1]. Let $X(t)$ be a second-order f.s.p. defined on $[a, b]$. For each finite partition Δ_n of $[a, b] : \Delta_n : a = t_0 < t_1 < \ldots < t_n = b$, and for arbitrary points $\acute{t_i}, t_{i-1} \leq \acute{t_i} \leq t_i, i = 1, 2, \ldots, n$, let $S_n = \sum_{i=1}^{n} \Delta t_i X(\acute{t_i})$ and $\mid \Delta_n \mid = \max_{1\leq i\leq n} \Delta t_i$, where $\Delta t_i = t_i - t_{i-1}$. Then the mean-square Riemann integral or m.s. integral of $X(t)$ on the interval $[a, b]$ is defined by

$$\int_a^b X(t)dt = \lim_{|\Delta_n|\to} S_n,$$

provided this limit exists and it is independent of the partition as well as the selected points $\acute{t_i}$.

At the same time, we call that $X(t)$ is m.s. integrable on $[a, b]$. If $X(t), t \in [a, b]$, is non-random, the m.s. convergence equals the convergence in D, this time the m.s. integral is called as R integral in D or DR integral. If $X(t)$ is mean-square continuous except for finitely many points of $[a, b]$ then $X(t)$ is mean-square integrable on $[a, b]$.

Theorem 2.12. [2]. *Let $X(t)$ and $Y(t)$ be m.s. integrable on $[a, b]$*

(i) *For each $\alpha \in [0, 1]$:*

$$[\int_a^b X(t)dt]^\alpha = \int_a^b [X(t)]^\alpha dt$$

(ii) *For each $\gamma, \beta \in R, \gamma X(t) + \beta Y(t)$ is m.s. integrable on $[a, b]$ and*

$$\int_a^b (\gamma X(t) + \beta Y(t))dt = \gamma \int_a^b X(t)dt + \beta \int_a^b Y(t)dt$$

(iii) *$X(t)$ is m.s. integrable on any subinterval of $[a, b]$, and*

$$\int_a^b X(t)dt = \int_a^c X(t)dt + \int_c^b X(t)dt, a \leq c \leq b.$$

(iv) *$EX(t)$ is DR integrable on $[a, b]$ and*

$$E\int_a^b X(t)dt = \int_a^b EX(t)dt.$$

(v) *If $\rho(X(t), Y(t))$ is classical Riemann integrable on $[a, b]$ then*

$$\rho(\int_a^b X(t)dt, \int_a^b Y(t)dt) \leq \int_a^b \rho(X(t), Y(t))dt.$$

Definition 2.13. [1]. A second-order f.s.p. $F(t), t \in T$, is m.s. differentiable at $t_0 \in T$ if there exists an $F'(t_0) \in L_2$ such that the m.s. limits

$$\lim_{h\to 0^+} \frac{F(t_0 + h) - F(t_0)}{h} \text{ and } \lim_{h\to 0^+} \frac{F(t_0) - F(t_0 - h)}{h}$$

exist and equal to $F'(t_0)$.
At the end points of T, we consider only the one-sided derivatives. If $F(t)$ is m.s. differential at every $t \in T$ then, we call $F(t)$ m.s. differentiable on T.

Theorem 2.14. [2]

(i) *If $X(t)$ is m.s. differentiable at $t_0 \in T$ then $X(t)$ is m.s. continuous at $t_0 \in T$.*
(ii) *If $X(t)$ and $Y(t)$ are m.s. differentiable on T, then for any $\gamma, \beta \in R, \gamma X(t) + \beta Y(t)$ is m.s. differentiable on T and $(\gamma X(t) + \beta Y(t))' = \gamma X'(t) + \beta Y'(t)$.*
(iii) *If $X(t)$, $t \in [a,b]$ is m.s. continuous then, the m.s. integral $Y(t) = \int_a^t X(s)ds$, $t \in [a,b]$ is m.s. differentiable and $Y'(t) = X(t)$.*
(iv) *If $X(t)$,$t \in T$ is m.s. differentiable then $[X(t)]^\alpha$ is m.s. differentiable and $[X'(t)]^\alpha = ([X(t)]^\alpha)'$, for all $\alpha \in [0,1]$. In the case of E^1, write $[X(t)]^\alpha = [\underline{X}^\alpha(t), \overline{X}^\alpha(t)]$, then $\underline{X}^\alpha(t)$ and $\overline{X}^\alpha(t)$ are m.s.differentiable and $[X'(t)]^\alpha = [\underline{X}^\alpha(t), \overline{X}^\alpha(t)]$.*
(v) *(Newton-Leibniz formula) If $X'(t)$ is m.s. integrable on $[a,b]$ then for each $t \in [a,b], X(t) = X(a) + \int_a^t X'(s)ds$.*

For above further details on mean-square calculus the reader is referred to [2].

3. Fuzzy stochastic differential equations (FSDE)

Let us first establish some definitions and notations on the fuzzy random vector. Let $X_1, \ldots, X_m$ be f.r.v.'s. $X = (X_1, \ldots X_m)$T is called an m-dimensional fuzzy random vector, where T denotes the transpose of the vector. It is a Borel measurable function $X : \Omega \to (E^n)^m = E^n \times \ldots \times E^n$. Let $L_2^m = \{X \mid X = (X_1, \ldots, X_m)^T X_i \in L_2, i = 1, 2, \ldots, m\}$. Define

$$\rho(X,Y) = \max_{1 \le i \le m} \rho(X_i, Y_i), \quad X, Y \in L_2^m$$

The norm $\| X \|_2$ of an element $X \in L_2^m$ is defined by

$$\| X \|_2 = \rho(X, \hat{0}) = \max_{1 \le i \le m} \| X_i \|^2 .$$

By the completeness of (L_2, ρ) and $Eq.$(2.1), $Eq.$(2.2) a standard proof applies that (L_2^m, ρ) is a complete metric space and ρ satisfies that

$$\rho(X+Z, Y+Z) = \rho(X,Y), \quad \rho(\lambda X, \lambda Y) = \mid \lambda \mid \rho(X,Y), \tag{3.1}$$

$$\rho(\lambda X, kY) \le \mid \lambda - k \mid \| X \|_2, \tag{3.2}$$

for any $X, Y, Z \in L_2^m$ and $\lambda, k \in \mathbb{R}$.

A second-order m-dimensional vector f.s.p. is characterized by a mapping of the interval T into L_2^m. For the sake of convenience, we shall adopt the notation $X(t) : T \to L_2^m$ in what follows. The m.s. continuity, m.s. differentiation, and m.s. integration associated with a second-order m-dimensional f.s.p. are defined with respect to the metric ρ in L_2^m. Hence, an m-dimensional f.r.p. $X(t), t \in T$, is m.s. continuous at t, for example, if $\rho(X(t+h), X(t)) \to 0$, as $h \to 0$. In view of this definition, it is clear that the m-dimensional f.s.p. $X(t)$ is m.s. continuous at $t \in T$ if and only if each of its component processes is m.s. continuous at t. Similar definitions and observations can be made with regard to m.s. differentiation and m.s. integration of the second-order m-dimensional f.s.p. $X(t)$.

We cosider fuzzy stochastic differential equations by

$$\begin{cases} X'(t) = F(t, X(t)), \quad t \in T = [t_0, b] \\ X(t_0) = X_0 \end{cases} \tag{3.3}$$

where F is a mapping: $T \times L_2^m \to L_2^m$ and $X_0 \in L_2^m$. We now consider the solution of $Eq.$(3.3) in the mean square sense. From Theorem 2 we know that $X(t)$ is a solution of $Eq.$(3.3) if and only if it is m.s. continuous and satisfies the integral equation

$$X(t) = X_0 + \int_{t_0}^{t} F(s, X(s))ds. \tag{3.4}$$

Theorem 3.1. [1] *Let F be m.s. continuous with respect to t and there exists a $k > 0$ such that*

$$\rho(F(t,X), F(t;Y)) \leq k\rho(X,Y) \tag{3.5}$$

for all $t \in T$ and $X, Y \in L_2^m$. Then $Eq.$(3.3) has a unique solution.

4. Characterization Theorem for the solutions of FSDEs by using ODEs

Theorem 4.1. *Let $F : T \to L_2^1$ be m.s. differentiable. Denote*

$$[F(t)]^\alpha = [\underline{F}^\alpha(t), \overline{F}^\alpha(t)], \quad \alpha \in [0,1].$$

Then the boundary functions $\underline{F}^\alpha(t), \overline{F}^\alpha(t)$ are m.s. differentiable

$$[F'(t)]^\alpha = [(\underline{F}^\alpha(t))', (\overline{F}^\alpha(t))'], \quad \alpha \in [0,1].$$

Proof. According to definition 12, we can defined a mapping $F : [t_0, T] \to L_2^1$ is m.s. differentiable at $t \in [t_0, T]$ if there exits a $F'(t) \in L_2^1$ such that the limits

$$\lim_{h\to 0}\left(\frac{F'(t+h) - F'(t)}{h}\right) = \lim_{h\to 0}\left(\frac{F'(t) - F'(t-h)}{h}\right)$$

exist and equal to $F'(t)$. Now

$$[F'(t+h) - F'(t)]^\alpha = [(\underline{F}^\alpha)'(t+h) - (\underline{F}^\alpha)'(t), (\overline{F}^\alpha)'(t+h) - (\overline{F}^\alpha)'(t)]$$

and similarly for $[F'(t) - F'(t-h)]^\alpha$. Dividing by h and passing to the limit gives the theorem. ☐

Let us consider the fuzzy stochastic differential equations initial value problem (FSDE)(IVP)

$$\begin{cases} x'(t) = f(t, x(t)), \\ x(t_0) = x_0. \end{cases} \tag{4.1}$$

where, $f : T \times L_2^m \to L_2^m$ and $x_0 \in L_2^m$. Then the above theorem shows us a way to translate the FSDE $Eq.$(4.1) into a system of ODE.

Let $[x(t)]^\alpha = [\underline{x}^\alpha(t), \overline{x}^\alpha(t)]$. Now, suppose $x(t)$ is m.s. differentiable according to Theorem 4.1, $[x'(t)]^\alpha = [(\underline{x}^\alpha(t))', (\overline{x}^\alpha(t))']$. Clearly, Eq.((4.1)) translates into the following system of ODEs

$$\begin{cases} (\underline{x}^\alpha(t))' = \underline{f}^\alpha(t, \underline{x}^\alpha(t), \overline{x}^\alpha(t)), \\ (\overline{x}^\alpha(t))' = \overline{f}^\alpha(t, \underline{x}^\alpha(t), \overline{x}^\alpha(t)), \\ \underline{x}^\alpha(0) = \underline{x}_0^\alpha, \\ \overline{x}^\alpha(0) = \overline{x}_0^\alpha. \end{cases} \tag{4.2}$$

where

$$[f(t,x)]^\alpha = [\underline{f}^\alpha(t, \underline{x}^\alpha(t), \overline{x}^\alpha(t)), \overline{f}^\alpha(t, \underline{x}^\alpha(t), \overline{x}^\alpha(t))].$$

In the following theorem we show that the FSDE $Eq.$(4.1) will be equivalent to system $Eq.$(4.2). The numerical solutions of the ODEs are extremely well studied in the literature, so any numerical method we can consider for the system of ODEs, since the solution will be as well as solution of the FSDE under study. We can use the numerical methods directly on the ODEs obtained by the following theorem.

Theorem 4.2. *Let us consider the FSDE Eq.*(4.1) *where* $f : T \times L_2^m \to L_2^m$ *is such that*

(i) $[f(t,x)]^\alpha = [\underline{f}^\alpha(t, \underline{x}^\alpha(t), \overline{x}^\alpha(t)), \overline{f}^\alpha(t, \underline{x}^\alpha(t), \overline{x}^\alpha(t))]$,

(ii) *there exist* $L > 0$ *such that*
$|\underline{f}^\alpha(t, \underline{x}(t), \underline{y}(t)), \underline{f}^\alpha(t, \overline{x}(t), \overline{y}(t))| \le L \max\{|\overline{x} - \underline{x}|, |\overline{y} - \underline{y}|\}$
and
$|\overline{f}^\alpha(t, \underline{x}(t), \underline{y}(t), \overline{f}^\alpha(t, \overline{x}(t), \overline{y}(t))| \le L \ \max\{|\overline{x} - \underline{x}|, |\overline{y} - \underline{y}|\}$ *for all* $\alpha \in [0,1]$.

Then the FSDE Eq.(4.1) *and the system of ODE Eq.*(4.2) *are equivalent.*

Proof. According to Theorems 2.12 and 2.14, $\underline{f}^\alpha$ and $\overline{f}^\alpha$ are the continuity of the function f. Further, the Lipschitz property in condition (2), we can show property as follows:

$$\max\{|\underline{f}^\alpha(t, \underline{x}(t), \underline{y}(t)), \underline{f}^\alpha(t, \overline{x}(t), \overline{y}(t))|, |\overline{f}^\alpha(t, \underline{x}(t), \underline{y}(t)), \overline{f}^\alpha(t, \overline{x}(t), \overline{y}(t))|\} \le L \ \max\{|\overline{x} - \underline{x}|, |\overline{y} - \underline{y}|\},$$

by the Hausdorff distance d property

$$\rho(x,y) = \max\{|\underline{x} - \underline{y}|, |\overline{x} - \overline{y}|\},$$

finally

$$\rho(f(t,x(t)), (f(t,y(t)) \le \rho(x,y). \tag{4.3}$$

According to Theorem 3.1, it shows FSDE $Eq.$(4.1) has a unique solution. By Theorem 4, we can show that the solution of FSDE are m.s. differentiable and so, implies the functions $(\overline{x}^\alpha)$ and $(\underline{x}^\alpha)$ are m.s. differentiable, and as a conclusion $((\overline{x}^\alpha), (\underline{x}^\alpha))$ is a solution of $Eq.$(4.2). Conversely. Let us suppose that we have a solution $((\overline{x}^\alpha), (\underline{x}^\alpha))$, with $\alpha \in [0,1]$ fixed, of the system $Eq.$(4.2). Also, the $Eq.$(4.3) implies the existence and uniqueness of the fuzzy stochastic differential solution x'. Now, since x is m.s. differentiable, $(\overline{x}^\alpha)$,$(\underline{x}^\alpha)$ the endpoints of $(x)^\alpha$ (which are obviously valid level sets of a fuzzy-valued function) is a solution of $Eq.$(4.2). Since the solution of $Eq.$(4.2) is unique, we have $(x)^\alpha$, that is the problems $Eq.$(4.1) and $Eq.$(4.2) are equivalent. □

5. Numerical Example

Example 5.1. Consider the following second-order linear fuzzy stochastic differential equation

$$Y''(t) = 5Y'(t) - 6Y(t) + \gamma(t), \qquad t \in [0,b] \tag{5.1}$$

If $\gamma(t)$, $t \in [0, \infty)$, is a Gaussian f.s.p., then $\gamma(t) = E(\gamma(t)) + \xi(t)$ (see [2]Theorem 4.2), where $\xi(t)$ is a real-valued Gaussian s.p. with $E(\xi(t)) = 0$ for all $t \ge 0$. Suppose that $E(\xi(t)) = u$, for all $t \geq 0$, u is a LR-fuzzy number, i.e.

$$u(y) = \begin{cases} L(\frac{b-y}{b-a}), & a \leq y < b \\ 1, & b \leq y \leq c \\ R(\frac{y-c}{d-c}), & c \leq y \leq d \\ 0, & \text{otherwise.} \end{cases}$$

where L, R :$[0,1] \to [0,1]$ are two fixed left-continuous and nonincreasing functions with $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$. Thus

$$[\gamma(t)]^\alpha = [f(\alpha) + \xi(t), g(r) + \xi(t)]$$
$$[\underline{\gamma(s)}]^\alpha + [\overline{\xi(s)}]^\alpha = f(\alpha) + g(\alpha) + 2 * \xi(t)$$
$$[\underline{\gamma(s)}]^\alpha - [\overline{\xi(s)}]^\alpha = f(\alpha) - g(\alpha)$$

where

$$f(\alpha) = b - (b-a)L^{-1}(\alpha), g(\alpha) = c + (d-c)R^{-1}(\alpha).$$

Hence the unique solution of $Eq.$(5.1) is

$$\begin{aligned} \underline{Y(t)}^\alpha = &(1/2) * (e^{2*t}(3,-1) + e^{3*t}(-2,1)(\underline{Y}_0^\alpha + \overline{Y}_0^\alpha) - (1/14) * (e^{6*t}(1,1) + e^{6*t}(1,1) \\ &+ e^{-t}(6,-1)(\underline{Y}_0^\alpha - \overline{Y}_0^\alpha) + \int_0^t (-e^{2*(t-s)} + e^{(t-s)})\xi(s)ds \\ &+ (f(\alpha) + g(\alpha)((1/6)(e^{3*t} - 1) - (1/4)(e^{2*t} - 1)) \\ &- (1/84)(f(\alpha) - g(\alpha))((1/84)(e^{6*t} - 1) - (1/14)(1 - e^{-t})) \\ \overline{Y(t)}^\alpha = &(1/2) * (e^{2*t}(3,-1) + e^{3*t}(-2,1)(\underline{Y}_0^\alpha + \overline{Y}_0^\alpha) + (1/14) * (e^{6*t}(1,1) + e^{6*t}(1,1) \\ &+ e^{-t}(6,-1)(\underline{Y}_0^\alpha - \overline{Y}_0^\alpha) + \int_0^t (-e^{2*(t-s)} + e^{(t-s)})\xi(s)ds \\ &+ (f(\alpha) + g(\alpha)((1/6)(e^{3*t} - 1) - (1/4)(e^{2*t} - 1)) \\ &- (1/84)(f(\alpha) + g(\alpha))((1/84)(e^{6*t} - 1) - (1/14)(1 - e^{-t})) \end{aligned}$$

6. **Conclusion**

The main of our purpose in the paper is solve fuzzy stochastic differential equation by using method ODEs. That, we prove theorem for the expression of concept, this theorem shows i) if f is be fuzzy stochastic function ii) it satisfy the Lipschitz condition. Then the FSDE $Eq.$(4.1) and the system of ODE $Eq.$(4.2) are equivalent. We can solve this system by Euler or Rung-Kutta method.

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