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Research Paper

A NEW ITERATIVE METHOD TO SOLVE THE ABSOLUTE VALUE EQUATION

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Abstract. This study presents a modified version of the Rohn iterative method to solve the absolute value equation in the form $Ax - |x| - b = 0$, where A is a matrix such that the norm of its inverse is less than 1. The proposed method converges linearly to the unique solution of the absolute value equation. It offers a low computational cost algorithm that provides an approximate solution with acceptable accuracy after only a few iterations. Furthermore, this study compares the structure and convergence rates of the proposed method, the generalized Newton method, and the standard Rohn method. The potential applications of the proposed method are demonstrated through a comparison with the generalized Newton and Rohn methods, using 100 randomly generated absolute value equations of various dimensions. In total, 900 problems are solved.

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Keywords: Absolute value equation, A modified Rohn iterative method, The generalized Newton method.

1. Introduction

We consider the absolute value equation (AVE)

$$Ax - |x| - b = 0, \tag{1.1}$$

where A is a given square matrix of order n, $x \in \mathbb{R}^n$, $|x| = (|x_1|, \ldots, |x_n|)^T$ and $b \in \mathbb{R}^n$ is a given vector. Mangasarian [7] presented the generalized Newton method to solve the AVE (1.1) in the situation that the singular values of A exceed 1. Hooshyarbakhsh et al. [14] suggested an iterative method to solve the AVE. Rohn [13] considering a special case of AVE proposed an algorithm to find the solution for it. Esmaeili, Mahmoodabadi, and Ahmadi [4] proposed a parametric uniform approximation method based on the Newton method to solve the AVE (1.1) which is globally convergent under some weaker conditions compared to the existing methods. Esmaeili, Mirzapour, and Mahmoodabadi [5] introduced a fast linearly convergent algorithm to solve the AVE under certain assumptions on A. There are some theoretical analyses of the AVE in the literature, which have presented some methods from different perspectives, e.g. the alternative theorems various equivalent formulations of the AVE, and the existence and nonexistence solutions. Mangasarian and Meyer [11] considered the AVE (1.1) where the singular values of A are greater than 1 and suggested a bilinear program to solve it. But Mangasarian and Meyer [11] and Mangasarian [8] have not reported any computational results. Mangasarian [9] presented some computational results

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for a linear-programming-based successive linearization algorithm utilizing a concave minimization model. Noor et al. [12] presented an iterative method to solve the AVE based on a minimization model and also discussed the convergence of their proposed method under suitable conditions. Mangasarian and Meyer [11] showed that the general NP-hard linear complementarity problem (LCP) [1, 2, 3], which consists of many mathematical programming problems, can be formulated as an AVE (1.1). This means that the general form of AVE (1.1) is NP-hard. Mangasarian [10] formulated the NP-hard n-dimensional knapsack feasibility problem as an equivalent AVE in an n-dimensional non-integer real variable space and proposed a finite succession of linear programs for solving the AVE. Yong [15] presented a smoothing method for the AVE in which the absolute value function was replaced by a smooth one, namely the aggregate function.

Mangasarian and Meyer [11] mentioned some cases concerning solvability, the existence and nonexistence of solutions as follows:

(i) For any $b \in \mathbb{R}^n$, the AVE is uniquely solvable, if the singular values of A exceed 1.
(ii) For any $b \in \mathbb{R}^n$, the AVE is uniquely solvable if $\|A^{-1}\| < 1$.
(iii) If $A \geq 0$, $\|A\| < 1$ and $b \leq 0$, then there is a non-negative solution to the AVE.
(iv) If $b < 0$ and $\|A\|_\infty \leq \frac{\mu}{2}$ where $\mu = \min_i |b_i| / \max_i |b_i|$, then the AVE has exactly 2^n distinct solutions, each of which has no zero components and a different sign pattern.
(v) If $0 \neq b \geq 0$ and $\|A\| < 1$, then the AVE has no solution.
(vi) If b has at least one positive element and $\|A\|_\infty < \dfrac{\bar{\mu}}{2}$ where

$$\bar{\mu} = \frac{\max_{b_i > 0} b_i}{\min_i |b_i|},$$

then the AVE has no solution,

where $\|.\|$ denotes the matrix 2-norm in this paper, unless otherwise stated. This study considers the conditions mentioned in the case of (ii). In this paper, we propose a low computational cost algorithm to solve the AVE (1.1) that is globally convergent under certain assumptions after a few iterations. One advantage of our proposed method is that it reduces the round-off errors and so this method can determine an approximate solution with the acceptable accuracy. In Section 2, the generalized Newton iterative method and the Rohn method are analyzed, followed by the introduction of a modified Rohn method. We prove that the modified Rohn method converges under weaker conditions and requires less CPU-time compared to both the generalized Newton method and the Rohn method, especially for large scale AVE. Computational results are presented in Section 3.

2. **The modified Rohn method**

Consider the sign function $sign(u)$, which for any vector u is defined as

$$(sign(u))_i = \begin{cases} 1 & u_i > 0 \\ 0 & u_i = 0 \\ -1 & u_i < 0 \end{cases}, \quad i = 1, \ldots, n,$$

and consider the following related diagonal matrix:

$$D(u) = diag(sign(u)).$$

The generalized Newton iterative method to solve the AVE (1.1) is as follows:

$$y^{i+1} = (A - D(y^i))^{-1} b, \tag{2.1}$$

where $(A - D(y^i))^{-1}$ is the inverse matrix of $(A - D(y^i))$ and the obtained approximate solutions y^i and y^{i+1} are related to the ith and $(i+1)$th iterations, respectively. For each $b \in \mathbb{R}^n$, Mangasarian [7] proved that the method (2.1) converges linearly to the unique solution of the AVE under condition $\|A^{-1}\| < \frac{1}{4}$ from any starting point y^0.

The Rohn iterative method to solve the AVE (1.1) is as follows:

$$z^{i+1} = A^{-1}(|z^i| + b), \tag{2.2}$$

where the obtained approximate solutions z^i and z^{i+1} are related to the ith and $(i+1)$th iterations, respectively. Hooshyarbakhsh et al. [14] proved that, if $\|A^{-1}\| < \frac{1}{2}$, then the method (2.2) converges linearly to the uniqe solution x^* of the AVE for each $b \in \mathbb{R}^n$.

We present the modified Rohn method to solve the AVE (1.1) as follows:

$$\begin{cases} w = x^i - S[x^i - A^{-1}(|x^i| + b)] \\ x^{i+1} = A^{-1}(|w| + b), \end{cases} \tag{2.3}$$

where S is a suitable matrix and the obtained approximate solutions x^i and x^{i+1} are related to the ith and $(i+1)$th iterations, respectively.
The method (2.2) is a special case of the method (2.3) in the case of $S = 0$. If $S = I$, then x^i in the method (2.3) is equal to z^{i+2} in the method (2.2). The method (2.3) reduces the round-off errors by using the parameter vector w, so the convergence rate and approximate solutions are improved. If $x^0 = 0$ is considered as a starting point, then the difference between the first iteration of the method (2.3) and the method (2.2) is $A^{-1}|SA^{-1}b|$, which means the approximate solutions are modified from the first iteration. At first, the algorithm (2.3) determines the LU decomposition of the matrix A which is used to solve two triangular systems in each iteration of the algorithm and so, the computational cost is reduced, which is very important when the matrix size is large. In the following, we show that the method (2.3) under the specific conditions converges linearly to the unique solution of the AVE from any starting point.

We seek for a matrix S such that $\|S\| = \|I - S\|$. Although there are some matrices that have this feature, we regard the diagonal form of S, which has less computational cost. We consider the matrix S as follows:

$$S = diag(s), \qquad s = (s_1, \ldots, s_n)^T \quad s.t \quad 0 < s_1 \leq s_2 \leq \cdots \leq s_n < 1. \tag{2.4}$$

Lemma 2.1. *Let S be from (2.4). If $s_1 + s_n = 1$, then $\|S\| = \|I - S\|$, $0 < s_1 \leq \frac{1}{2}$, and $\frac{1}{2} \leq s_n < 1$.*

Proof. Notice that

$$\|S\| = \max_{1 \leq i \leq n} s_i = s_n,$$

$$\|I - S\| = \max_{1 \leq i \leq n} (1 - s_i) = 1 - \min_{1 \leq i \leq n} s_i = 1 - s_1.$$

If $s_1 + s_n = 1$, then $s_n = 1 - s_1$. Thus, it results $\|S\| = \|I - S\|$. Since $s_1 + s_n = 1$ and $0 < s_1 \leq s_n < 1$, so $0 < s_1 \leq \frac{1}{2}$ and $\frac{1}{2} \leq s_n < 1$. □

Consider the matrix S based on assumptions of lemma 2.1 for the rest of this paper.

Lemma 2.2. *Let x and y be vectors in $\mathbb{R}^n$, then*

$$\||x| - |y|\| \leq \|x - y\|. \tag{2.5}$$

Proof. For any $x_i, y_i \in \mathbb{R}$, we have $||x_i| - |y_i|| \leq |x_i - y_i|$. Therefore

$$\||x| - |y|\|^2 = \sum_{i=1}^{n} ||x_i| - |y_i||^2 \leq \sum_{i=1}^{n} |x_i - y_i|^2 = \|x - y\|^2.$$

Thus, we can conclude the inequality (2.5). □

The following theorem proves the convergence of the method (2.3).

Theorem 2.3. *If $\|A^{-1}\| < 1$, then the method (2.3) converges linearly to the unique solution x^* of the AVE from any starting point x^0 for each $b \in \mathbb{R}^n$.*

Proof. According to (2.3), we have $Ax^{i+1} = |w| + b$. On the other hand, x^* is the unique solution of the AVE, therefore $Ax^* = |x^*| + b$. So,

$$A(x^{i+1} - x^*) = |w| + b - |x^*| - b = |w| - |x^*|.$$

Regarding the above equality and lemmas 2.1 and 2.2, and the equation $x^* - A^{-1}|x^*| = A^{-1}b$, we have:

$$\begin{aligned}
\|x^{i+1} - x^*\| &= \|A^{-1}(|w| - |x^*|)\| \leq \|A^{-1}\|\|w - x^*\| \\
&= \|A^{-1}\|\|x^i - S(x^i - A^{-1}(|x^i| + b)) - x^*\| \\
&= \|A^{-1}\|\|(x^i - x^*) - S(x^i - A^{-1}(|x^i| + b))\| \\
&= \|A^{-1}\|\|(x^i - x^*) - S(x^i - A^{-1}|x^i| - A^{-1}b)\| \\
&= \|A^{-1}\|\|(x^i - x^*) - S(x^i - A^{-1}|x^i| - x^* + A^{-1}|x^*|)\| \\
&= \|A^{-1}\|\|(x^i - x^*) - S(x^i - x^*) + SA^{-1}(|x^i| - |x^*|)\| \\
&= \|A^{-1}\|\|(I - S)(x^i - x^*) + SA^{-1}(|x^i| - |x^*|)\| \\
&\leq \|A^{-1}\| \left(\|(I - S)(x^i - x^*)\| + \|SA^{-1}\|\|x^i - x^*\|\right) \\
&\leq \|A^{-1}\| \left(\|I - S\|\|x^i - x^*\| + \|S\|\|A^{-1}\|\|x^i - x^*\|\right) \\
&= \|A^{-1}\|\|S\| \left(1 + \|A^{-1}\|\right) \|x^i - x^*\|.
\end{aligned}$$

Letting $\gamma = \|A^{-1}\|$ result in

$$\|x^{i+1} - x^*\| \leq \gamma\|S\|(1 + \gamma)\|x^i - x^*\| = c\|x^i - x^*\|.$$

If $c < 1$, then the method (2.3) converges. So, we determine γ and $\|S\|$ such that $c < 1$:

$$\gamma\|S\|(1 + \gamma) < 1 \longrightarrow \|S\| < \frac{1}{\gamma(1 + \gamma)}.$$

Since $\|A^{-1}\| < 1$, then $\frac{1}{2} < \frac{1}{\gamma(1+\gamma)}$. Also, we notice that if $0 < \gamma < \frac{\sqrt{5}-1}{2}$, then $1 < \frac{1}{\gamma(1+\gamma)}$ and if $\frac{\sqrt{5}-1}{2} < \gamma < 1$, then $\frac{1}{2} < \frac{1}{\gamma(1+\gamma)} < 1$. On the other hand, we have $\frac{1}{2} \leq \|S\| < 1$, thus for $\frac{\sqrt{5}-1}{2} < \gamma < 1$ we consider S such that $\frac{1}{2} \leq \|S\| < \frac{1}{\gamma(1+\gamma)}$ and for $0 < \gamma < \frac{\sqrt{5}-1}{2}$ we can determine S such that $\frac{1}{2} \leq \|S\| < 1$. Furthermore, if $0 < \gamma < 1$ and S is determined based on above conditions, then the method (2.3) converges linearly to the unique solution x^* of the AVE from any starting point x^0 for any $b \in \mathbb{R}^n$. □

Since the method (2.1) is a well-known method to solve the AVE (1.1), so we compare the method (2.3) which is a modified Rohn method with methods (2.1) and (2.2).
In each iteration of the method (2.1), the diagonal matrix $D(y^i)$ is computed and then y^{i+1} is determined by the MATLAB command $(A - D(y^i)) \setminus b$. As it is shown in the following, although the method (2.1) finds the solution in fewer iterations than the method (2.3), it has higher CPU-time and computational cost than the method (2.3). According to lemma 6 and proposition 7 in [7], if $\|A^{-1}\| < 1$, then the asymptotic error constant of the method (2.1) is not less than 1. This means that, under this condition, the convergence of the method (2.1) is not validated. However, if $\|A^{-1}\| < 1$, then the value of c is less than 1 and so the method (2.3) converges. If we consider the common condition $\|A^{-1}\| < \frac{1}{4}$, then both methods (2.1) and (2.3) converge to the unique solution of the AVE. In this case, the value of c is less than $\frac{5}{16}$, whereas the asymptotic error constant of the method (2.1) is less than 1. Hence, we expect that the method (2.3) attains the solution sooner than the method (2.1).

On the other hand, this study modifies the method (2.2) which has less computational cost than the method (2.1) to achieve an approximate solution with high accuracy. Also, as the method (2.3) has fewer iterations than the method (2.2), computational cost of the method (2.3) is less than that of the method (2.2) for large scale AVE. In the next section, we illustrate this claim according to CPU-time of each method. In other words, the method (2.3) is convergent under weak conditions and has less CPU-time than methods (2.1) and (2.2).

3. **Computational results**

This section presents the obtained results of the method (2.1) (GN), the method (2.2) (Rhn), and the method (2.3) (MR), which have been obtained by MATLAB on AMD E2-1800 APU with 1.70 GHz. We consider 100 random square matrices for each size 200, 300, and 400. Overall, 900 problems are solved through examples. The square matrix A and the exact solution x^* are created randomly by

$$\begin{aligned} A &= 20 * randn(n,n) - 20 * randn(n,n), \\ x^* &= randn(n,1) - randn(n,1), \end{aligned}$$

and then we set

$$b = Ax - |x|,$$

for each run of the algorithms. The average CPU-time for methods is denoted by ACPU. Also, we consider AIter as the averaged number of iterations of methods. A residual error,

$$\|Ax - |x| - b\| < \epsilon,$$

is used as a stop condition, where $\epsilon = 10^{-6}$ and $\epsilon = 10^{-8}$ are the bounds of error. The averaged residual error of solutions obtained by methods is denoted by ARes.
In order to achieve the computational cost in each method, we consider the number of flops [6], where a flop is a floating point add, subtract, multiply, or divide. The computational cost of the method (2.1) in one iteration is

$$\frac{2}{3}n^3 + 4n^2 + 5n,$$

where n is the size of matrices and vectors. Suppose that i is the number of the iterations for each method. The total computational cost of the method (2.1) is as follows:

$$(\frac{2}{3}n^3 + 4n^2 + 5n) \times i.$$

The total computational cost of the method (2.2), which has two parts, is as follows:

$$\frac{2}{3}n^3 + (4n^2 + 5n) \times i.$$

Also, the total computational cost of the method (2.3), which has two parts, is as follows:

$$\frac{2}{3}n^3 + (6n^2 + 9n) \times i.$$

The average total computational cost of methods is denoted by ATCC.

Example 3.1. In this example, the matrix A is considered such that the norm of its inverse matrix is less than $\frac{1}{4}$. Given this assumption, we obtain the average results from 100 random square matrices of orders 200 with the average norm of inverse matrix 0.14046, 300 with the average norm of inverse matrix 0.13675 , and 400 with the average norm of the inverse matrix 0.16224 which are summarized in Table 1, where $\epsilon = 10^{-6}$.

Table 1. Comparison among GN, Rhn, and MR when $\|A^{-1}\| < \frac{1}{4}$.

Methods ($n = 200$)	ACPU	AIter	ATCC	ARes
GN	$4.5043e-02$	$2.4000e+00$	$1.3186e+07$	$1.6221e-10$
Rhn	$2.8278e-02$	$5.2400e+00$	$6.1770e+06$	$6.2410e-08$
MR	$2.3962e-02$	$3.0533e+00$	$6.0716e+06$	$9.7375e-08$
Methods ($n = 300$)	ACPU	AIter	ATCC	ARes
GN	$7.6988e-02$	$2.4810e+00$	$4.5555e+07$	$4.0444e-10$
Rhn	$6.0693e-02$	$5.0886e+00$	$1.9840e+07$	$5.5293e-08$
MR	$4.0775e-02$	$3.0253e+00$	$1.9642e+07$	$1.0450e-07$
Methods ($n = 400$)	ACPU	AIter	ATCC	ARes
GN	$1.1938e-01$	$2.6721e+00$	$1.1573e+08$	$7.0276e-10$
Rhn	$1.0151e-01$	$5.0984e+00$	$4.5940e+07$	$8.4106e-08$
MR	$6.7479e-02$	$3.0164e+00$	$4.5573e+07$	$8.5613e-08$

Example 3.2. The matrix A is considered such that the norm of its inverse matrix is greater than $\frac{1}{4}$ and less than $\frac{1}{2}$. According to this assumption, we obtain the average results from 100 random square matrices of orders 200 with the average norm of inverse matrix 0.36857, 300 with the average norm of inverse matrix 0.38526, and 400 with the average norm of inverse matrix 0.38154 which are reported in Table 2, where $\epsilon = 10^{-6}$.

Example 3.3. We apply methods GN, Rhn, and MR for 100 randomly generated absolute value equations in the presence of the square matrices in which the norm of their inverse matrices is greater than $\frac{1}{2}$ and less than 1. Also in this case all random matrices are square matrices of orders 200, 300, and 400 with the average norm of inverse matrix 0.72205, 0.70084, and 0.68869 respectively. The obtained average results are summarized in Table 3, where $\epsilon = 10^{-6}$.

Example 3.4. In this example, we illustrate the results of methods for one randomly generated absolute value equation with size 500, where $\|A^{-1}\| = 0.30545 < \frac{\sqrt{5}-1}{2}$ and $\epsilon = 10^{-8}$. The results of this example are reported in Table 4 and Figure 1.

TABLE 2. Comparison among GN, Rhn, and MR when $\frac{1}{4} \leq \|A^{-1}\| < \frac{1}{2}$.

Methods $(n = 200)$	ACPU	AIter	ATCC	ARes
GN	$4.9788e-02$	$2.8022e+00$	$1.5396e+07$	$8.1087e-11$
Rhn	$2.4327e-02$	$6.0110e+00$	$6.3011e+06$	$1.2527e-07$
MR	$2.1372e-02$	$3.5934e+00$	$6.2022e+06$	$9.7618e-08$

Methods $(n = 300)$	ACPU	AIter	ATCC	ARes
GN	$7.6048e-02$	$2.8118e+00$	$5.1628e+07$	$2.2256e-10$
Rhn	$4.9788e-02$	$5.9294e+00$	$2.0143e+07$	$1.2506e-07$
MR	$4.0033e-02$	$3.4588e+00$	$1.9877e+07$	$1.2527e-07$

Methods $(n = 400)$	ACPU	AIter	ATCC	ARes
GN	$1.2870e-01$	$2.7738e+00$	$1.2013e+08$	$3.3124e-10$
Rhn	$1.0168e-01$	$5.8810e+00$	$4.6442e+07$	$1.1931e-07$
MR	$7.1699e-02$	$3.4524e+00$	$4.5993e+07$	$9.2235e-08$

TABLE 3. Comparison among GN, Rhn, and MR when $\frac{1}{2} \leq \|A^{-1}\| < 1$.

Methods $(n = 200)$	ACPU	AIter	ATCC	ARes
GN	$3.0055e-01$	$3.0000e+00$	$1.6483e+07$	$4.1259e-11$
Rhn	$1.1368e-01$	$7.3049e+00$	$6.5094e+06$	$1.4904e-07$
MR	$1.1223e-01$	$4.8171e+00$	$6.4981e+06$	$1.4181e-07$

Methods $(n = 300)$	ACPU	AIter	ATCC	ARes
GN	$3.3511e-01$	$3.0000e+00$	$5.5084e+07$	$1.2975e-10$
Rhn	$1.5607e-01$	$7.0417e+00$	$2.0546e+07$	$1.1186e-07$
MR	$1.3755e-01$	$4.6667e+00$	$2.0532e+07$	$1.0010e-07$

Methods $(n = 400)$	ACPU	AIter	ATCC	ARes
GN	$3.2243e-01$	$2.8730e+00$	$1.2443e+08$	$2.1464e-10$
Rhn	$2.0750e-01$	$6.7143e+00$	$4.6977e+07$	$1.3519e-07$
MR	$1.5924e-01$	$4.3333e+00$	$4.6842e+07$	$1.3136e-07$

TABLE 4. Comparison among GN, Rhn, and MR when $\|A^{-1}\| < \frac{\sqrt{5}-1}{2}$.

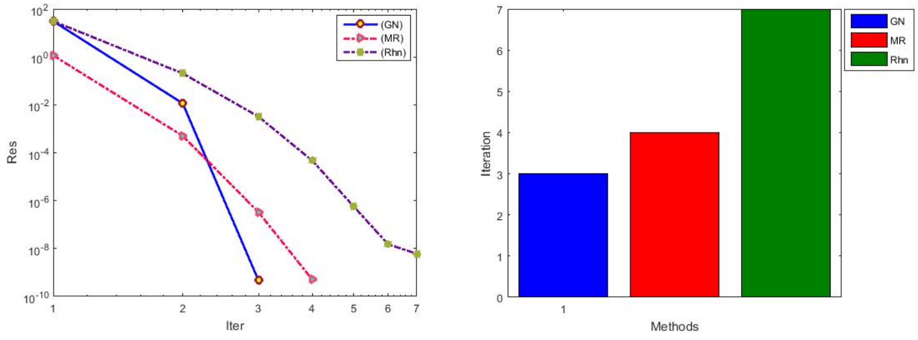
Methods $(n = 500)$	ACPU	AIter	ATCC	ARes
GN	$8.4377e-01$	3	$2.5300e+08$	$2.3294e-10$
Rhn	$4.0823e-01$	7	$9.0351e+07$	$3.0117e-09$
MR	$3.1635e-01$	4	$8.9351e+07$	$2.5330e-10$

Example 3.5. In this example, we illustrate the results of methods for one randomly generated absolute value equation with size 500, where $\frac{\sqrt{5}-1}{2} < \|A^{-1}\| = 0.64824 < 1$ and $\epsilon = 10^{-8}$. In this example, the method (2.2) (Rhn) is divergent. The results of this example are reported in Table 5.

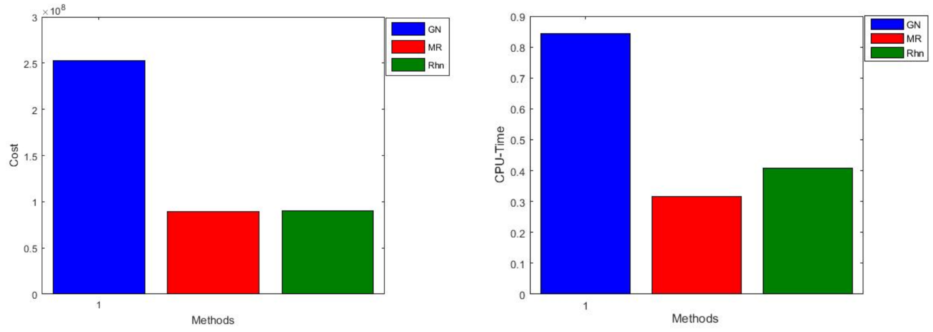
TABLE 5. Comparison among GN, Rhn, and MR when $\frac{\sqrt{5}-1}{2} < \|A^{-1}\| < 1$.

Methods $(n = 500)$	ACPU	AIter	ATCC	ARes
GN	$5.7850e-01$	3	$2.5300e+08$	$3.0733e-10$
Rhn	$-$	$-$	$-$	$-$
MR	$2.5539e-01$	5	$9.0856e+07$	$5.5228e-10$

It is clear that the CPU-time of the method (2.3) is less than that of methods (2.1) and (2.2) and it has acceptable accuracy compared to the accuracy of methods (2.1) and (2.2).



(A) $Res = \|Ax - |x| - b\|$ (B) Iteration



(C) Total computational cost (D) CPU-time of methods

Figure 1. Results of Example 3.4

According to the results reported in tables, we can conclude that the method (2.3) has less computational cost than methods (2.1) and (2.2) for large scale AVE and it is the main advantage of the method (2.3).

4. Conclusion

This study proposed a linearly convergent modified Rohn method to solve the NP-hard absolute value equation $Ax - |x| = b$ under certain assumptions on A. The method (2.3) can work under the less stringent condition in which the norm of the inverse of the matrix A is less than 1. Also, the method (2.3) is well defined and it works better than methods (2.1) and (2.2) in terms of CPU-time, the computational cost, and the convergence condition on A. We can change the matrix S to obtain a new modification of the method (2.2) for further research.

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