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Research Paper

AUTOCOMMUTATIVITY DEGREE FOR TOPOLOGICAL GROUPS

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Abstract. Let G be a compact Hausdorff topological group and suppose that $\mathrm{Aut}(G)$ be the group of topological automorphisms of G, which itself is a compact Hausdorff topological group. In this paper, we will define the notion of autocommutativity degree for the group G, which generalizes the concept of autocommutativity degree, in the case of finite groups. We will prove some properties of the autocommutativity degree for topological groups. In particular, we will state an upper bound for the autocommutativity degree of non-abelian groups and investigate the structure of groups that attain this upper bound. Also we provide some examples of inifinite topological groups and their autocommutativity degree.

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1. **Introduction**

In finite group theory, there are many counting problems such as the commutativity degree, relative commutativity degree, autocommutativity degree, and numerous other problems whose definitions inherently rely on the order of the group and its subgroups. Consequently, examining these concepts in the case of infinite groups becomes problematic. A natural question that arises is whether these notions can be extended in a way that makes them applicable to infinite groups as well. Naturally, such an extension should be constructed so that, in the special case of finite groups, it coincides with the original definitions and the corresponding results.

For the first time, Gustafson in [8] defined the commutativity degree (originally defined for finite groups) using the concept of measure for compact topological groups and demonstrated that analogous results to the finite case hold for such groups. In [6, 3, 4] a similar approach was taken to study the relative commutativity degree for compact topological groups.

Another counting problem in finite group theory is the autocommutativity degree, which represents the probability that a group element remains fixed under a group automorphism. More precisely, let G be a finite group and $\mathrm{Aut}(G)$ be its automorphism group. The autocommutativity degree of the group G is denoted by $\mathrm{Pr}_A(G)$ and is defined as follows

$$\mathrm{Pr}_A(G) = \frac{|D|}{|G||\,\mathrm{Aut}(G)|},$$

where

$$D = \{(x, \alpha) \in G \times \mathrm{Aut}(G) \mid \alpha(x) = x\}.$$

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As is evident from the above definition, this concept is based on the order of the group, which cannot be expressed in the infinite case. In this paper, inspired by [8], we will define this concept for a class of topological groups and prove results analogous to those in the finite case. The proposed definitions are constructed in such a way that they fully coincide with the conventional definitions in the finite case, and the results for finite groups can be viewed as special cases of these more general statements.

Let G be a compact Hausdorff topological group. Then, according to Sections 8 and 9 of Chapter II in [5], there exists a left Haar measure μ on the σ-algebra of Borel sets with the property that $\mu(xE) = \mu(E)$ for every element $x \in G$. By imposing the normalization condition $\mu(G) = 1$, we may assume that μ is the unique probability measure on G. Let $\mathrm{Aut}(G)$ be the set of all topological automorphisms of G (continuous automorphisms with continuous inverse), then it is known that $\mathrm{Aut}(G)$ with open-compact topology is a compact Hausdorff topological group. Assume that ν is the left Haar measure on $\mathrm{Aut}(G)$ with normalization condition $\nu(\mathrm{Aut}(G)) = 1$. We consider the product measure $\mu \times \nu$ on the space $G \times \mathrm{Aut}(G)$. In section 3 we use this product measure to define autocommutativity degree for topological groups.

2. Preliminaries

In this section, we introduce the key concepts that will be used throughout the paper. In this paper, G will always denote a compact Hausdorff topological group, and $\mathrm{Aut}(G)$ will represent its group of topological automorphisms. Also μ and ν are the normalized Haar measure on G and $\mathrm{Aut}(G)$ respectively.

If G is a group and H be a subgroup of G, the index of H in G will be denoted by $|G : H|$. The center of the group G is denoted by $Z(G)$ and consists of elements that commute with every element of G.

Theorem 2.1 (Proposition 1.3.5 from [7])**.** *Let G be a group and H, K be subgroups of G such that $K \leq H \leq G$. Then we have*

$$|G : K| = |G : H| \cdot |H : K|.$$

For a fixed element $a \in G$, the set of automorphisms in $\mathrm{Aut}(G)$ that fix a is called the stabilizer (or centralizer) of a and is denoted by $C_A(a)$, i.e.

$$C_A(a) = \{\alpha \in \mathrm{Aut}(G) \mid \alpha(a) = a\}.$$

It is obvious that $C_A(a)$ is a subgroup of $\mathrm{Aut}(G)$. For any automorphism $\alpha \in \mathrm{Aut}(G)$, the set of elements in G that are fixed by α is called the fixed-point subgroup of α and is denoted by $\mathrm{Fix}(\alpha)$, that is

$$\mathrm{Fix}(\alpha) = \{x \in G \mid \alpha(x) = x\}.$$

We can easily check that $\mathrm{Fix}(\alpha)$ is a subgroup of G.

The absolute center $L(G)$ (see [1]) consists of elements fixed by all automorphisms of G, that is

$$L(G) = \{a \in G \mid \alpha(a) = a,\ \forall \alpha \in \mathrm{Aut}(G)\}.$$

Remark 2.2. Let G be a group and $Inn(G)$ be the set of inner automorphisms of G. If $\phi_a \in Inn(G)$, then $\phi_a(g) = a^{-1}ga$, so an element x is fixed by ϕ_a if and only if a and x commute. This shows that if an element x is fixed by all elements of $Inn(G)$, then x commutes with every element of G, i.e. it is an element of $Z(G)$. Since every element of $L(G)$

is fixed by all elements of $\mathrm{Aut}(G)$, it is fixed by every inner automorphism and therefore become an element of $Z(G)$. Hence always we have $L(G) \subseteq Z(G)$.

Lemma 2.3 (Lemma 2.2 from [6])**.** *Let G be a group and H a subgroup of G. Then*

$$\mu(H) = \begin{cases} \frac{1}{|G:H|} & \text{if } |G:H| < \infty \\ 0 & \text{if } |G:H| = \infty \end{cases}$$

where $|G:H|$ denotes the index of H in G. We will use the above theorem in several places without direct reference.

From the proof of Theorem 1 from Section 2 of [8], we have the following lemma that will be used in the proof of some theorems.

Lemma 2.4. *Let χ_D be the characteristic function on D. Then*

$$\int_{\mathrm{Aut}(G)} \chi_D(x, y)\, d\nu(y) = \nu(C_A(x)).$$

3. Main results

Throughout the remaining of this paper, G will always denote a compact Hausdorff topological group, and $\mathrm{Aut}(G)$ will represent its group of topological automorphisms, which itself is a compact Hausdorff topological group. Also μ and ν are the normalized Haar measure on G and $\mathrm{Aut}(G)$, respectively. As mentioned in the introduction, the autocommutativity degree for a finite groups G, is defined as follows

$$\mathrm{Pr}_A(G) = \frac{|D|}{|G||\mathrm{Aut}(G)|},$$

where

$$D = \{(x, \alpha) \in G \times \mathrm{Aut}(G) \mid \alpha(x) = x\}. \tag{3.1}$$

For infinite cases, this definition is useless because of the order of group, but measure is the key concept that will help us to define autocommutativity degree for infinite topological groups.

Define the function $f : G \times \mathrm{Aut}(G) \to G$ by

$$f(x, \alpha) = x^{-1}\alpha(x),$$

then f is a continuous function and $D = f^{-1}(1)$. Consequently, D is a closed set and therefore measurable.

Definition 3.1. The autocommutativity degree of the groups G, is denoted by $\mathrm{Pr}_A(G)$ and is defined as follows

$$\mathrm{Pr}_A(G) = \mu \times \nu(D) = \int_{G\times\mathrm{Aut}(G)} \chi_D(x, \alpha)\, d\mu(x)\, d\nu(\alpha)$$

where D is the subset defined in equation 3.1 and χ_D is the characteristic function on D.

The autocommutativity degree of a group shows the probability that a random element of G is fixed by a random element of $\mathrm{Aut}(G)$. Before stating some properties of this definition, in the next remark we note that above definition for autocommutativity degree, coincide with this concept for finite groups.

Remark 3.2. If G is finite, then G and $\operatorname{Aut}(G)$ with discrete topology are topological groups and the Haar measures reduces to the normalized counting measures

$$\mu(x) = \frac{1}{|G|}, \quad \nu(\alpha) = \frac{1}{|\operatorname{Aut}(G)|}.$$

So by definition of autocommutativity degree for topological groups we have

$$\operatorname{Pr}_A(G) = \sum_{x \in G} \sum_{\alpha \in \operatorname{Aut}(G)} \frac{1}{|G| \cdot |\operatorname{Aut}(G)|} \chi_D(x, \alpha) = \frac{|D|}{|G| \cdot |\operatorname{Aut}(G)|}.$$

Thus, the topological definition coincides with the definition in finite case. Therefore the definition of autocommutativity degree for finite groups is a special case of our general definition for topological groups.

Lemma 3.3. *Always we have*

$$\operatorname{Pr}_A(G) = \int_G \nu\left(C_A(x)\right) d\mu(x) = \int_{\operatorname{Aut}(G)} \mu(\operatorname{Fix}(\alpha)) d\nu(\alpha).$$

Proof. By definition of autocommutativity degree we have

$$\operatorname{Pr}_A(G) = \mu \times \nu(D) = \int_{G \times \operatorname{Aut}(G)} \chi_D(x, \alpha)\, d\mu(x)\, d\nu(\alpha),$$

Since both measures are σ-finite and χ_D is measurable and non-negative, we apply Fubini's theorem and we gain

$$\operatorname{Pr}_A(G) = \int_{G \times \operatorname{Aut}(G)} \chi_D(x, \alpha)\, d\mu(x)\, d\nu(\alpha) = \int_G \left(\int_{\operatorname{Aut}(G)} \chi_D(x, \alpha)\, d\nu(\alpha) \right) d\mu(x).$$

Since for a fixed element $x \in G$ we have

$$\{\alpha \in \operatorname{Aut}(G) \;\mid\; (x, \alpha) \in D\} = C_A(x),$$

therefore by lemma 2.4,

$$\int_{\operatorname{Aut}(G)} \chi_D(x, \alpha) d\nu(\alpha) = \nu(C_A(x)).$$

Hence, we obtain the first equality

$$\operatorname{Pr}_A(G) = \int_G \nu\left(C_A(x)\right) d\mu(x).$$

Again by using the definition and Fubini's theorem while reversing the order of integration, we obtain

$$\operatorname{Pr}_A(G) = \int_{G \times \operatorname{Aut}(G)} \chi_D(x, \alpha)\, d\mu(x)\, d\nu(\alpha) = \int_{\operatorname{Aut}(G)} \left(\int_G \chi_D(x, \alpha)\, d\mu(x) \right) d\nu(\alpha).$$

For fixed α, we have

$$\{x \in G \;\mid\; (x, \alpha) \in D\} = \operatorname{Fix}(\alpha),$$

therefore the inner integral becomes

$$\int_G \chi_D(x, \alpha) d\mu(x) = \mu(\operatorname{Fix}(\alpha)),$$

thus we establish the second equality

$$\Pr_A(G) = \int_{\mathrm{Aut}(G)} \mu(\mathrm{Fix}(\alpha)) d\nu(\alpha).$$

□

Since $\Pr_A(G)$ is a probability, it is obvious that $\Pr_A(G) \leq 1$ for any group G. In the next theorem we classify groups that the upper bound occurs.

Theorem 3.4. *Let G be a compact Hausdorff topological group, then $\Pr_A(G) = 1$ if and only if $G \cong \mathbb{Z}_2$ or G is trivial.*

Proof. If G be trivial or $G \cong \mathbb{Z}_2$, then $\mathrm{Aut}(G) = \{id\}$, hence by lemma 3.3, we have

$$\Pr_A(G) = \int_{\mathrm{Aut}(G)} \mu\left(\mathrm{Fix}(\alpha)\right)\right) d\nu(x) = \nu(\{\mathrm{id}\}) \cdot 1 = 1,$$

and the result follows.

Now suppose that $\Pr_A(G) = 1$. We claim that $\mathrm{Aut}(G)$ is trivial. Let

$$X = \{\alpha \in \mathrm{Aut}(G) \mid \alpha \text{ is non-trivial}\}.$$

Using lemma 3.3, autocommutativity degree of G can be computed as follows

$$\begin{aligned}
\Pr_A(G) &= \int_{\mathrm{Aut}(G)} \mu\left(\mathrm{Fix}(\alpha)\right)\right) d\nu(x) \\
&= \int_{\mathrm{id}} \mu\left(G)\right) d\nu(x) + \int_X \mu\left(\mathrm{Fix}(\alpha)\right)\right) d\nu(x) \\
&= \nu(\{\mathrm{id}\}) \cdot 1 + \int_X \mu\left(\mathrm{Fix}(\alpha)\right) d\nu(x)
\end{aligned}$$

Since $\mathrm{Fix}(\alpha)$ is a subgroup of G, for any element of X, we have $\mathrm{Fix}(\alpha) < G$, therefore $|G : \mathrm{Fix}(\alpha)| \geq 2$ which means that $\mu(\mathrm{Fix}(\alpha)) \leq \frac{1}{2}$. It follows that

$$\Pr_A(G) = \nu(\{\mathrm{id}\}) \cdot 1 + \int_X \mu\left(\mathrm{Fix}(\alpha)\right) d\nu(x) \leq \nu(\{\mathrm{id}\}) + \frac{1}{2}\nu(X),$$

if $\nu(X) > 0$, since $X \cup \{\mathrm{id}\} = \mathrm{Aut}(G)$, $\Pr_A(G) < 1$ which is a contradiction, so $\nu(X) = 0$. Also $X \cup \{\mathrm{id}\} = \mathrm{Aut}(G)$ with $\nu(X) = 0$ results that $\mathrm{Aut}(G) = \{\mathrm{id}\}$ and we are done.

Now if G is finite, $\mathrm{Aut}(G) = \{\mathrm{id}\}$ implies that $G \cong \mathbb{Z}_2$. If G is infinite then it must be trivial because every non-trivial group has non-trivial automorphsim, in fact if G is non-abelian then conjugation by any non-central element is a non-trivial automorphism and if G is abelian then $x \mapsto x^{-1}$ is non-trivial except that $G \cong \bigoplus_{i=1}^{\infty} \mathbb{Z}_2$ which in this case $\mathrm{Aut}(G)$ is not trivial. □

If G is a non-abelian group, then its autocommutativity degree is far from above upper bound. The next theorem shows this bound for non-abelian groups.

Theorem 3.5. *Let G be a non-abelian group. Then $\Pr_A(G) \leq \frac{5}{8}$.*

Proof. By lemma 3.3, we have

$$\Pr_A(G) = \int_G \nu\left(C_A(x)\right) d\mu(x).$$

Suppose that $x \in G$. If $x \in L(G)$, then by definition of absolute center, x is fixed by all elements of $\mathrm{Aut}(G)$ and this means that $C_A(x) = \mathrm{Aut}(G)$, hence we have $\nu(C_A(x)) = 1$. If

$x \notin L(G)$, then $C_A(x) < \mathrm{Aut}(G)$. So $|\mathrm{Aut}(G) : C_A(x)| \geq 2$ and by lemma 2.3, $\nu(C_A(x)) \leq \frac{1}{2}$. Therefore we have

$$\begin{aligned}\mathrm{Pr}_A(G) &= \int_G \nu\left(C_A(x)\right) d\mu(x) \\ &= \int_{L(G)} \nu\left(C_A(x)\right) d\mu(x) + \int_{G-L(G)} \nu\left(C_A(x)\right) d\mu(x) \\ &\leq \int_{L(G)} d\mu(x) + \int_{G-L(G)} \frac{1}{2} d\mu(x) \\ &= \mu(L(G)) + \frac{1}{2}\mu(G - L(G)) \\ &= \frac{1}{2}\mu(L(G)) + \frac{1}{2}.\end{aligned}$$

We know that if $G/Z(G)$ is cyclic, then G is abelian. Therefore, if G is non-abelian, then $G/Z(G)$ is non-cyclic, and thus $|G : Z(G)| \geq 4$, which implies that $\mu(Z(G)) \leq \frac{1}{4}$. On the other hand, by remark 2.2, $L(G) \leq Z(G)$, hence we have $\mu(L(G)) \leq \frac{1}{4}$. Combining this with above equation we gain

$$\mathrm{Pr}_A(G) \leq \frac{1}{2}\mu(L(G)) + \frac{1}{2} \leq \frac{1}{2}\frac{1}{4} + \frac{1}{2} = \frac{5}{8}.$$

Therefore for every non-abelian group G, we have $\mathrm{Pr}_A(G) \leq \frac{5}{8}$. □

We note that for finite non-abelian groups, the same inequality applies to the autocommutativity degree. Furthermore, analogous results (as detailed in the next theorem) hold for finite non-abelian groups where the upper bound is attained.

Theorem 3.6. *Let G be a non-abelian group such that $\mathrm{Pr}_A(G) = \frac{5}{8}$, then*

$$\frac{G}{Z(G)} \cong \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Proof. Similar to the proof of theorem 3.5, we have

$$\mathrm{Pr}_A(G) = \int_G \nu\left(C_A(x)\right) d\mu(x) \leq \int_{L(G)} d\mu(x) + \int_{G-L(G)} \frac{1}{2} d\mu(x).$$

Also, $\mu(L(G)) \leq \frac{1}{4}$ and for elements $x \in G - L(G)$, $\nu(C_A(x)) \leq \frac{1}{2}$. If $\mu(L(G)) < \frac{1}{4}$ or $\nu(C_A(x)) < \frac{1}{2}$, then a simple computation shows that $\mathrm{Pr}_A(G) < \frac{5}{8}$ which is a contradiction. So we have $\mu(L(G)) = \frac{1}{4}$ and $\nu(C_A(x)) = \frac{1}{2}$ for almost all $x \in G - L(G)$. Since $L(G) \leq Z(G)$, $\mu(L(G)) \leq \mu(Z(G)$. If $\mu(L(G)) < \mu(Z(G))$ then $|G : Z(G)| < 4$ which force $\frac{G}{Z(G)}$ to be cyclic and G must be abelian which is a contradiction. So $\mu(Z(G)) = \frac{1}{4}$ and $|\frac{G}{Z(G)}| = 4$. If $\frac{G}{Z(G)}$ is cyclic, again we conclude that G must be abelian which contradicts with our hypothesis, hence $\frac{G}{Z(G)}$ is not cyclic and therefore it is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$. □

Now we state two example to complete the discussion. We mention that any finite example satisfy our definition, but here we provide two infinite topological group and compute their autocommutativity degree.

Example 3.7. Suppose that

$$C = \{e^{i\theta} \;\big|\; \theta \in \mathbb{R}\},$$

then, under the ordinary multiplication of complex numbers and the usual topology of complex numbers, C is a compact Hausdorff topological group. In this example we compute autocommutativity degree for this group. First note that $\mathrm{Aut}(C)$ is isomorphic to $\mathbb{Z}_2$, in fact C has exactly two group automorphisms, one is the identity and the other is complex conjugation which we denote them by id and σ respectively. If $\alpha = \mathrm{id}$ then every element of C is fixed by α, so we have $\mathrm{Fix}(\mathrm{id}) = C$. If α is the complex conjugation, then only 1 and -1 is fixed by α, hence $\mathrm{Fix}(\sigma) = \{1, -1\}$. Since $\mu(\{1, -1\}) = 0$ and $\mathrm{Aut}(C) \cong \mathbb{Z}_2$, by using lemma 3.3, we have

$$\begin{aligned}\mathrm{Pr}_A(G) &= \int_{\mathrm{Aut}(G)} \mu(\mathrm{Fix}(\alpha)) d\nu(\alpha) \\ &= \nu(id)\mu(\mathrm{Fix}(id)) + \nu(\sigma)(\mathrm{Fix}(\sigma)) \\ &= \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2}.\end{aligned}$$

For many parts of the following example we used [2].

Example 3.8. Let $G = \mathrm{SO}(3)$ be the group of rotations in $\mathbb{R}^3$, equipped with its normalized Haar measure μ. It is known that the automorphism group of SO(3) is isomorphic to SO(3) itself, in fact every automorphism of SO(3) is an inner automorphism and since $Z(\mathrm{SO}(3)) = 1$ we have

$$\mathrm{SO}(3) \cong \frac{\mathrm{SO}(3)}{Z(\mathrm{SO}(3))} \cong Inn(\mathrm{SO}(3)) = \mathrm{Aut}(\mathrm{SO}(3)).$$

Now we compute the autocommutativity degree of SO(3). Using lemma 3.3, the autocommutativity degree is given by

$$\mathrm{Pr}_A(\mathrm{SO}(3)) = \int_{\mathrm{Aut}(\mathrm{SO}(3))} \mu(\mathrm{Fix}(\alpha))\, d\nu(\alpha),$$

where α is an inner automorphism. If α is the inner automorphism induced by an element $P \in \mathrm{SO}(3)$, then fixed points of α is as follows

$$\mathrm{Fix}(\alpha) = \{A \in \mathrm{SO}(3) \mid PAP^{-1} = A\} = C_{\mathrm{SO}(3)}(P).$$

For $P \neq I$, the centralizer $\mathrm{Fix}(\alpha)$ is isomorphic to SO(2), which has μ-measure zero in SO(3). For $P = I$, $\mathrm{Fix}(\alpha) = \mathrm{SO}(3)$ with $\mu(\mathrm{SO}(3)) = 1$. Since $\{I\}$ has ν-measure zero in $\mathrm{Aut}(\mathrm{SO}(3))$, we conclude that

$$\mathrm{Pr}_A(\mathrm{SO}(3)) = \nu(\{I\}) \cdot 1 + \int_{P \neq I} 0\, d\nu(\alpha) = 0.$$

Therefor, $\mathrm{Pr}_A(\mathrm{SO}(3)) = 0$.

4. Conclusion

In this paper, we introduced and analyzed the autocommutativity degree $\mathrm{Pr}_A(G)$ for compact Hausdorff topological groups, generalizing the concept from finite groups to a measure-theoretic framework. By leveraging Haar measures on G and $\mathrm{Aut}(G)$, we established foundational properties of $\mathrm{Pr}_A(G)$ and demonstrated its alignment with the classical definition in the finite case. We proved that for non-abelian compact Hausdorff groups, $\mathrm{Pr}_A(G) \leq \frac{5}{8}$, which mirrors the well-known bound in finite group theory. We showed that the equality $\mathrm{Pr}_A(G) = 1$ holds only for trivial groups or $G \cong \mathbb{Z}_2$, while groups attaining the upper bound $\frac{5}{8}$ satisfy $G/Z(G) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, thus extending important structural results from finite groups to the topological groups. Also with explicit computations we demonstrated that $\Pr_A(C) = \frac{1}{2}$ for the circle group C and $\Pr_A(\mathrm{SO}(3)) = 0$.

This work provides a natural bridge between discrete and continuous group theory by extending probabilistic group invariants to topological settings. The measure-theoretic approach developed here suggests several promising directions for future research, including potential applications to ergodic theory, operator algebras, and the study of more general classes of topological groups.

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