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Research Paper

TWO-STAGE NETWORK DEA MODEL UNDER INTERVAL DATA

FATEMEH SADAT SEYED ESMAEILI, MOHSEN ROSTAMY-MALKHALIFEH*, AND FARHAD HOSSEINZADEH LOTFI

Abstract. The main goal of this paper is to propose interval network data envelopment analysis (INDEA) model for performance evaluation of network decision making units (DMUs) with two stage network structure under data uncertainty. It should be explained that for dealing with uncertainty of data, an interval programming method as a popular uncertainty programming approach is applied. Also, to show the applicability of proposed model, INDEA approach is implemented for performance measurement and ranking of 10 insurance companies from Iranian insurance industry. Note that insurance companies are undoubtedly one of the most important pillars of the financial markets, whose great performance will drive the economy of the country. The empirical results indicate that the proposed INDEA is capable to be utilized to assess the performance of two-stage DMUs in the presence of interval data.

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1. introduction

Network data envelopment analysis (NDEA) is an applicable mathematical programming approach for performance assessment and ranking of homogenous decision making units (DMUs) with network structure such as two-stage, series, parallel, etc. [1, 2, 3, 4, 5]. Nowadays, NDEA approach is wieldy used in many real-world application and case studies such as insurance companies, bank branches, hospitals, hotels, supply chains, airports, airlines, etc. [6, 7].

It is important to consider the uncertainty in the data that used to evaluate the performance of DMUs more accurately, since uncertainty is an integral part of the real-world. Also, one of the most important assumptions in DEA is that the measured data are certain and conclusive. However, a little bias or deviation in the values of data can cause significant differences in final results [8, 9, 10, 11]. As a result, it is necessary to develop an uncertain network data envelopment analysis (UNDEA) method that is capable to be used under uncertainty environment.

Therefore, the purpose of this study is to present an uncertain network data envelopment analysis in order to performance evaluation of DMUs considering the internal structure and data uncertainty. To achieve this purpose, a two-stage DEA model and interval programming

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*Corresponding author.

approach will be applied in order to propose the interval network data envelopment analysis (INDEA) model.

It should be noted that the interval programming approach is wieldy applied in DEA to calculate the interval efficiency under uncertainty. Finally, the proposed INDEA model will be implemented in Iranian insurance industry. Note that the internal structure of insurance companies (ICs) can be considered as a two-stage network system including the marketing and investment processes. The rest of this paper is organized as follows: The theoretical backgrounds of the two-stage DEA model will be explained in section 2. Then, an interval network data envelopment analysis model will be proposed in section 3. Application of proposed interval two-stage DEA approach will be examined using real data of 10 insurance companies in Iran. Finally, conclusions and discussions of current study will be introduced in section 5.

2. **Two-Stage DEA Model**

Suppose that there are n network $DMUs$ with a two-stage structure that each $DMU_j(1,...,n)$ has I inputs $x_{ij}(i = 1,2,\dots,I)$ in stage 1, D intermediate (linking) variables $z_{ij}(d = 1,2,\dots,D)$ and R outputs $y_{rj}(r = 1,2,\dots,R)$ in stage 2. Chen and Zhou [12] proposed a two-stage DEA approach by integrating the envelopment form of the first and second stages. The extended two-stage network DEA approach based on study of Chen and Zhou [12] for DMU under investigation (DMU_0) is as Model (2.1):

$$
Min\ \theta/\varphi
$$

$$
s.t\ \begin{cases}
\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{i0}, \forall i, \\
\sum_{j=1}^{n} \lambda_j z_{dj} \ge \widehat{z}_{d0},\ \forall d, \\
\sum_{j=1}^{n} \lambda_j = 1, \\
\theta \le 1, \\
\lambda_j \ge 0,\ \forall j, \\
\sum_{j=1}^{n} \mu_j z_{dj} \le \widehat{z}_{d0},\ \forall d, \\
\sum_{j=1}^{n} \mu_j y_{rj} \ge \varphi y_{r0},\ \forall r, \\
\sum_{j=1}^{n} \mu_j = 1, \\
\varphi \ge 1, \\
\mu_j \ge 0,\ \forall j.
\end{cases} \tag{2.1}
$$

It should be noted that $\widehat{z}_{d0}$ which is presented in Model (1) is a decision variable. Also, the model is introduced under variable returns to scale assumption.

3. **Interval Two-Stage DEA Model**

In this section, an uncertain network DEA model for performance evaluation of network DMUs with two-stage structure under uncertainty will be proposed. It should be noted all data have interval uncertainties $x_{ij} \in \left[x_{ij}^l, x_{ij}^u\right]$, $z_{dj} \in \left[z_{dj}^l, z_{dj}^u\right]$ and $y_{rj} \in \left[y_{rj}^l, y_{rj}^u\right]$ where only the upper and lower bounds are known. Due to the uncertainty in the data, the interval data envelopment analysis (IDEA) approach proposed by Despotis and Smirlis[13] will be applied. According to the interval programming approach, Models (3.1) and (3.2)are used to calculate the lower and upper bounds of the efficiency scores of DMU_0 , respectively:

$$\xi^l = Min\ \theta/\varphi$$
$$(3.1)\qquad s.t \begin{cases} \sum_{j=1}^{n} \lambda_j x_{ij}^l + \lambda_0 x_{i0}^u \leq \theta x_{i0}^u, \forall i, \\ \sum_{j=1}^{n} \lambda_j z_{dj}^u \geq \widehat{z}_{d0},\ \forall d, \\ \sum_{j=1}^{n} \lambda_j = 1, \\ \theta \leq 1, \\ \lambda_j \geq 0,\ \forall j, \\ \sum_{j=1}^{n} \mu_j z_{dj}^l \leq \widehat{z}_{d0},\ \forall d, \\ \sum_{j=1}^{n} \mu_j y_{rj}^u + \mu_0 y_{r0}^l \geq \varphi y_{r0}^l,\ \forall r, \\ \sum_{j=1}^{n} \mu_j = 1, \\ \varphi \geq 1, \\ \mu_j \geq 0,\ \forall j, \end{cases}$$

and

$$\xi^u = Min\ \theta/\varphi$$
$$(3.2)\qquad s.t \begin{cases} \sum_{j=1}^{n} \lambda_j x_{ij}^u + \lambda_0 x_{i0}^l \leq \theta x_{i0}^l, \forall i, \\ \sum_{j=1}^{n} \lambda_j z_{dj}^l \geq \widehat{z}_{d0},\ \forall d, \\ \sum_{j=1}^{n} \lambda_j = 1, \\ \theta \leq 1, \\ \lambda_j \geq 0,\ \forall j, \\ \sum_{j=1}^{n} \mu_j z_{dj}^u \leq \widehat{z}_{d0},\ \forall d, \\ \sum_{j=1}^{n} \mu_j y_{rj}^l + \mu_0 y_{r0}^u \geq \varphi y_{r0}^u,\ \forall r, \\ \sum_{j=1}^{n} \mu_j = 1, \\ \varphi \geq 1, \\ \mu_j \geq 0,\ \forall j. \end{cases}$$

It should be explained that the efficiency of DMUs based on interval network data envelopment analysis (INDEA) are obtained in an interval form.

4. Application and Implementation

In this section, the proposed research approach is implemented for performance assessment of 10 Iranian insurance companies. It should be noted that inputs 1 and 2 are respectively operation costs and insurance costs, intermediate measures 1 and 2 are respectively direct written premium and reinsurance premium and finally outputs 1 and 2 are underwriting profit and investment profit, respectively. Now, the results of INDEA model for calculating stage 1, stage 2, and overall efficiency of insurance companies are presented in Tables 1, 2 and 3, respectively: As shown in Table 3, from the first stage (marketing process) perspectives,

Insurance Companies	First Stage	
	Lower	Upper
IC 01	0.57	0.61
IC 02	0.61	0.98
IC 03	0.54	0.55
IC 04	0.75	0.82
IC 05	0.62	0.62
IC 06	1.00	1.00
IC 07	0.72	0.75
IC 08	1.00	1.00
IC 09	1.00	1.00
IC 10	1.00	1.00

TABLE 1. The Results of INDEA-First Stage

Insurance Companies	Second Stage	
	Lower	Upper
IC 01	0.58	0.61
IC 02	1.00	1.00
IC 03	1.00	1.00
IC 04	0.60	0.62
IC 05	1.00	1.00
IC 06	0.40	0.42
IC 07	1.00	1.00
IC 08	0.36	0.38
IC 09	0.53	0.55
IC 10	0.12	0.13

TABLE 2. The Results of INDEA-Second Stage

IC 06, IC 08, IC 09, and IC 10 have the best performance. Also, from the second stage (investment process) perspectives, IC 02, IC 03, IC 05, and IC 07 have the best performance. Finally, IC 07, IC 05, and IC 02 are the best insurance companies from overall perspective, respectively.

Insurance Companies	Overall	
	Lower	Upper
IC 01	0.33	0.37
IC 02	0.61	0.98
IC 03	0.54	0.55
IC 04	0.45	0.51
IC 05	0.62	0.62
IC 06	0.40	0.42
IC 07	0.72	0.75
IC 08	0.36	0.31
IC 09	0.53	0.55
IC 10	0.12	0.13

Table 3. The Results of INDEA-Overall

5. **Conclusions**

In this study, an uncertain network DEA approach for performance assessment of two-stage DMUs under interval data is proposed. It should be explained that to deal with data uncertainty, an interval programming approach is applied. Also, the applicability of the proposed interval two-stage DEA is demonstrated by a case study from Iranian insurance Industry. Note that ignoring the internal structure of network DMUs, along with data uncertainty, may lead to invalid results. Accordingly, the implementation of INDEA model is necessary in real-world applications.

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(Fatemeh Sadat Seyed Esmaeili) Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.
Email address: rozita.esmaeeli@gmail.com

(Mohsen Rostamy-Malkhalifeh) Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.
Email address: mohsen_rostamy@yahoo.com

(Farhad Hosseinzadeh Lotfi) Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran
Email address: farhad@hosseinzadeh.ir