Mathematical Analysis
& Convex Optimization

Vol. 2 (2021), No. 2, 45–55
https:// maco.lu.ac.ir
DOI: 10.52547/maco.2.2.5

Research Paper

THE SENSITIVITY ANALYSIS AND SUSTAINABILITY RADIUS OF ECONOMIC EFFICIENCY IN DATA ENVELOPMENT ANALYSIS

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Abstract. In economics, a production function relates the outputs of a production process to the inputs of the production. Generally, the production function is not available due to the complexity of the production process, the changes in production technology. Therefore, we have to consider an approximation of the production function. Data Envelopment Analysis (DEA) is a non-parametric methodology for obtaining an approximation of the production function and assessing the relative efficiency of economic units. Sensitivity analysis and sustainability evaluation of Decision Making Units (DMUs) are as the most important concerns of Decision Makers (DM). This study considers the sustainability radius of economic performance of DMUs and then proposes some approaches combined with sensitivity analysis for determining the sustainability radius of cost efficiency, revenue efficiency and profit efficiency of units. The proposed approaches eliminate the unit under evaluation from the observed data and disturb the data of it, based on the sensitivity analysis, to determine the sustainability radius of cost efficiency, revenue efficiency and profit efficiency of decision making units. Potential application of our proposed methods is illustrated with a dataset consisting of 21 medical centers in Taiwan.

MSC(2010): 90B10.
Keywords: Data Envelopment Analysis, Economic efficiency, Cost efficiency, Revenue efficiency, Profit efficiency, Sustainability radius.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric methodology for assessing the relative efficiency of Decision Making Units (DMUs) with multiple inputs and multiple outputs (Charnes et al. [7], Banker et al. [2], Färe et al. [13], Zhu [36]) It assigns an efficiency measure between 0 and 1 to each unit. The larger the efficiency score, the better2 performance the unit under evaluation has. A DMU is efficient if its efficiency score is equal to 1, otherwise it is inefficient. The original DEA models consider the situation where unit price and unit cost information are not available, or where their uses are limited because of variability in the prices and costs. However, DEA can be used to evaluate the different types of efficiency of DMUs, such as cost efficiency, revenue efficiency and profit efficiency of units when the information on prices and costs are known exactly.

The cost efficiency (CE) can be interpreted as the ability of each decision making unit with multiple inputs and multiple outputs to produce the current outputs at minimal cost. Farrell [14] introduced the concept of CE in the situation that the input and output values and input

Date: Received: June 16,2021, Accepted: December 1, 2021.
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prices are known exactly. Färe [13] proposed methods that present empirical implementations of the cost efficiency measures in DEA. The problem of the measuring the cost efficiency of units has attracted attentions of several scholars. See, Cooper et al. [9] , Sueyoshi [30], Tone [33], Tone and Sahoo [34, 35], Maniadakis and Thanassoulis [22], Sengupta and Sahoo [29], Jahanshahloo et al. [18], Sahoo et al. [26], Sahoo et. al. [27], Sarkar [4], Mirdehghan et al. [23], Ghiyasi [16], Cesaroni [4], Tohidnia and Tohidi [31] among others. Camanho and Dyson [3] and Fang and Li [10, 11, 12] evaluated the cost efficiency of units in the presence of data uncertainty. Kuosmanen and Post [20, 21] proposed models to determine the cost efficiency of units in the situation that the input prices are uncertain. Toloo et al. [32] considered the cost efficiency of units in the presence of interval data. Cherchye et al. [8] considered the cost efficiency analysis of research programs in economics and business management faculties. Mostafaee and Saljooghi [24] considered two scenarios for assessing the cost efficieny of DMUs. The first scenario evaluated the cost efficiency of units in the presence of data uncertainty and the second scenario assessed the cost efficiency of DMUs in the situation that both data and input prices were uncertain.

On the other hand, the revenue efficiency can be interpreted as the ability of each decision making unit with multiple inputs and multiple outputs to consume the current inputs at the maximal revenue. Mozaffari et al. [25] evaluated the cost efficiency and revenue efficiency of DMUs in DEA-R models. Salehpour and Aghayi [28] considered the revenue efficiency of units in the case of data uncertainty Johnes and Ruggiero [19] assessed the revenue efficiency of higher education institutions in UK in 2012- 2013. Khoshgova and RostamyMalkhalifeh considered the cost efficiency of units in the presence of integer data and in the absence of the convexity principle in the production technology. The distance function has3 been used as a metric concept for calculating the cost efficiency by Sahoo et al. (2014), Cherchye et al. [8] and Chambers et al. [5, 6]. Ghyasi [16] applied inverse DEA to evaluate the cost and revenue efficiency of units. Ahangaria and Rostamy-Malkhalifeh [1] considered the profit inefficiency and the cost inefficiency of units, respectively.

Given the importance of the sensitivity analysis in DEA, this paper focuses on the sensitivity and stability analysis and proposes some models to determine the sustainability radius of the cost efficiency, revenue efficiency and profit efficiency of DMUs. The rest of this paper is organized as follows: section 2 proposes some models to determine the minimal cost, maximal revenue and maximal profit of units. Section 3 suggests models to determine the sustainability radius of the cost, revenue and profit efficiencies. A numerical example and a case study reported in Mozaffari et al. [25] are applied to illustrate the potential application of our proposed methods. The last section concludes the paper.

2. The proposed methods to evaluate the economic efficiency

Consider a system of DMUs, denoted by $DMU_j, j = 1, 2, \dots, n$, where each unit consumes m different inputs to generate different outputs. The ith input and rth output for DMU_j are denoted by x and y_{rj}, respectively, for $I = 1, 2, \dots, m$ and $r = 1, 2, \dots, s$. Also, suppose that C and R are the vectors of input costs and the vector of output prices, respectively.

This section focuses on determining the minimal cost, maximal revenue and maximal profit of DMU_o in the absence of the unit under evaluation. For this purpose, we eliminate DMU_o from the observed data and then formulate model (2.1) to evaluate this unit and so, we can determine the minimal cost to produce the current outputs of the eliminated unit.

$$z_1^* = \min Cx$$
$$s.t. \begin{cases} \sum_{j=1, j\neq o}^{n} \lambda_j x_j \leq x, \\ \sum_{j=1, j\neq o}^{n} \lambda_j y_j \geq y, \\ \lambda_j \geq 0, \;\; x \geq 0. \end{cases} \tag{2.1}$$

Suppose that x^* is an optimal solution of model (2.1). Therefore, Cx^* shows the minimal cost to produce y_0 in the absence of DMU_o.
Similarly, we consider the maximal revenue of the unit under evaluation that can be obtained by consuming the current inputs of DMU_o. For this purpose, we eliminate the unit under evaluation from the observed data and then formulate model (2.2) to evaluate this unit and so, we can determine the maximal revenue that can be obtained by consuming the current inputs of the eliminated unit.

$$z_2^* = \max Ry$$
$$s.t. \begin{cases} \sum_{j=1, j\neq o}^{n} \lambda_j x_j \leq x_o, \\ \sum_{j=1, j\neq o}^{n} \lambda_j y_j \geq y, \\ \lambda_j \geq 0, \;\; y \geq 0. \end{cases} \tag{2.2}$$

Suppose that y^* is an optimal solution for model (2.2). Therefore, Ry^* shows the maximal revenue that can be obtained by consuming x_o in the absence of DMU_o. Finally, we eliminate the unit under evaluation from the observed data and formulate model (2.3) to determine the maximal profit that can be obtained by consuming x_o and producing y_o in the absence of DMU_o.

$$z_3^* = \max Ry - Cx$$
$$s.t. \begin{cases} \sum_{j=1, j\neq o}^{n} \lambda_j x_j \leq x, \\ \sum_{j=1, j\neq o}^{n} \lambda_j y_j \geq y, \\ x \leq x_o, \; y \leq y_o, \\ \lambda_j \geq 0, \;\; x \geq 0, \; y \geq 0. \end{cases} \tag{2.3}$$

Suppose that x^*, y^* is an optimal solution for model (3), Therefore, $Ry^* - Cx^*$ shows the maximal profit can be obtained in the absence of DMU_o.

3. The proposed models to determine the sustainability radius of economic efficiency

In this section, we propose some models to determine the sustainability radius of the cost efficiency, the revenue efficiency and the profit efficiency of the units by using the optimal solutions of models (2.1), (2.2) and (2.3), respectively. For this purpose, we consider the predetermined vectors and propose models to determine the maximum possible movement along these directions such that the cost efficiency, the revenue efficiency and the profit efficiency do not change. Hence, the movement vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}$, $g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and

$g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ are defined and the inputs and the outputs of DMU_o are disturbed along these directions for determining the sustainability radius of the cost, revenue and profit efficiencies.

3.1. **The sustainability radius of the cost efficiency.** In this section, we consider the direction vectors $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$, $i = 1, 2, 3$, and determine the maximum possible movement along these direction vectors such that the cost efficiency of DMU_o does not change and introduce the sustainability radius of the cost efficiency of DMU_o. Hence, model (3.1) is formulated as follows:

$$\theta^* = \max\ \theta$$
$$(3.1) \qquad s.t. \begin{cases} \sum_{j=1, j\neq o}^{n} \lambda_j x_j \leq x_0 + \theta g_x, \\ \sum_{j=1, j\neq o}^{n} \lambda_j y_j \geq y_0 + \theta g_y, \\ C(x_0 + \theta g_x) \leq Cx^*, \\ \lambda_j \geq 0,\ \theta \text{ is free.} \end{cases}$$

Where x^* is an optimal solution of model (2.1). Model (3.1) determines the step length θ such that the cost efficiency of DMU_o does not change along the directions $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}$, $i = 1, 2, 3$. model (4) is solved for three direction vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}$, $g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ and the minimum amount of θ^*, obtained by considering these direction vectors, is introduced as the sustainability radius of the cost efficiency of DMU_o.

3.2. **The sustainability radius of the revenue efficiency.** In this section, we consider the direction vectors $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}$, $i = 1, 2, 3$. and determine the maximum possible movement along these direction vectors such that the revenue efficiency of DMU_0 does not change and introduce the sustainability radius of the revenue efficiency of DMU_0. Hence, model (3.2) is formulated as follows:

$$\beta^* = \max\ \beta$$
$$(3.2) \qquad s.t. \begin{cases} \sum_{j=1, j\neq o}^{n} \lambda_j x_j \leq x_0 + \beta g_x, \\ \sum_{j=1, j\neq o}^{n} \lambda_j y_j \geq y_0 + \beta g_y, \\ R(x_0 + \beta g_x) \leq Ry^*, \\ \lambda_j \geq 0,\ \beta \text{ is free.} \end{cases}$$

where Y^* is an optimal solution of model (2.2). Model (3.2) determines the step length β^* such that the revenue efficiency of DMU_o does not change along the directions $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}$, $i = 1, 2, 3$. Now, model (3.2) is solved for three direction vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}$, $g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ and the minimum amount of β, obtained by considering these direction vectors, is introduced as the sustainability radius of the revenue efficiency of DMU_o.

3.3. **The sustainability radius of the profit efficiency.** In this section, we consider the direction vectors $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}, i = 1, 2, 3.$ and determine the maximum possible movement along these direction vectors such that the profit efficiency of DMU_o does not change and introduce the sustainability radius of the profit efficiency of DMU_o. Hence, model (3.3) is formulated as follows:

$$\rho^* = \max\ \rho$$
$$(3.3) \qquad s.t. \begin{cases} \sum_{j=1, j\neq o}^{n} \lambda_j x_j \leq x_0 + \rho g_x, \\ \sum_{j=1, j\neq o}^{n} \lambda_j y_j \geq y_0 + \rho g_y, \\ R(x_0 + \beta g_x) \leq R y^*, \\ C(x_0 + \rho g_x) \leq C x^*, \\ \lambda_j \geq 0,\ \rho \text{ is free.} \end{cases}$$

where (x^*, y^*) is an optimal solution of model (2.3). Model (3.3) determines the step length ρ^* such that the profit efficiency of DMU_o does not change along the directions $g_i = \begin{pmatrix} g_x \\ g_y \end{pmatrix}, i = 1, 2, 3$. Now, model (3.3) is solved for three direction vectors $g_1 = \begin{pmatrix} -x_o \\ 0 \end{pmatrix}$, $g_2 = \begin{pmatrix} 0 \\ y_o \end{pmatrix}$ and $g_3 = \begin{pmatrix} -x_o \\ y_o \end{pmatrix}$ and the minimum amount of rho^*, obtained by considering these direction vectors, is introduced as the sustainability radius of the profit efficiency of DMU_o.

4. Numerical examples

In this section, the proposed models are illustrated in a numerical example with five DMUs and a case study, reported in Mozaffari et al. (2014), with 21 medical centers in Taiwan.

Example 4.1. Consider five decision making units. Each DMU consumes two inputs to produce two output. Table 1 reports the data of units. Table 2 shows the cost of inputs and the output prices for all DMUs. In this example, the vector of input costs and also, the vector of output prices are not the same for all DMUs. Now, we apply the proposed approaches to determine the sustainability radius of the cost efficiency, the revenue efficiency and the profit efficiency of units. Hence, model (2.1) and model (3.1) are solved and the results are reported in Table 3. The second and the third columns of Table 3 show the first and the second inputs obtained by model (2.1), respectively. The fourth column of this table reports the optimal value of model (2.1) and the fifth column of Table 3 shows the sustainability radius of the cost efficiency of DMUs.Then, we solve models (2.2) and (3.2) and report the results in Table 4. The second and the third columns of this table show the first and the second outputs obtained by model (2.2). The fourth column shows the optimal value of model (2.2) and the fifth column reports the sustainability radius of the revenue efficiency of units.Finally, we solve models (2.3) and (3.3) and report the results in Table 5. The second and the third columns of this table show the first and the second inputs and the fourth and the fifth columns of Table 5 report the first and the second outputs obtained by model (2.3). The sixth column shows the optimal value of model (2.3) and the seventh column reports the sustainability radius of the profit efficiency of units.

DMU	x_{1j}	x_{2j}	y_{1j}	y_{2j}
1	12	0.21	138	21
2	10	0.1	143	28
3	4	0.16	157	21
4	19	0.12	158	21
5	14	0.06	157	28

TABLE 1. The data of DMUs in Example 1

DMU	C_1	C_2	R_1	R_2
1	100	50	10	30
2	110	40	9	27
3	105	42	8	25
4	107	50	9	29
5	111	47	10	28

TABLE 2. The input costs and output prices

DMU	x_1^*	x_2^*	z_1^*	θ^*
1	4	0.16	408.0	0.8671
2	5.33	0.21	595.2	1.0000
3	10.97	0.11	1157.4	0.0000
4	8.13	0.12	0.4294	0.7319
5	5.33	0.21	602.0	0.4662

TABLE 3. The results of models (1) and (4)

DMU	y_1^*	y_2^*	z_2^*	β^*
1	259.05	38.43	3743.4	0.5520
2	157	25.2	2093.4	0.0506
3	57.2	11.2	737.0	0.0000
4	243.35	41.51	3393.94	0.4546
5	85.8	16.8	1328.4	0.0000

TABLE 4. The results of models (2) and (5)

Example 4.2. In this example, the results of applying our proposed approaches to the data set in Mozaffari et al. (2014) are presented. This data set has 21 decision making units which are medical centers in Taiwan with two inputs, The number of sickbeds (x_1) and the number of physicians (x_2) in order to produce three outputs, the total number of out-patients (y_1),

DMU	x_1^*	x_2^*	y_1^*	y_2^*	z_3^*	ρ^*
1	12	0.21	259.05	38.43	2532.9	0.5520
2	4.65	0.15	152	32.3	204.6	0.1054
3	4	0.16	157	21	0.4362	0.0000
4	19	0.12	243.35	41.51	1354.94	0.4546
5	6,12	0.21	74.2	19.4	704.51	0.0000

TABLE 5. The results of models (1) and (4)

the number of in-patients (y_2) and the total number of surgeries (y_3). The input /output data are reported in Table 6.

Suppose that the vector of input costs is $(45, 70)$ and the vector of output prices is $(10, 18, 126)$. In this example, the vector of input costs and the vector of output prices are the same for all DMUs. Now, we apply our proposed methods to determine the sustainability radius of the cost, the revenue and the profit efficiencies for this data set. Hence, model (2.1) and model (3.1) are solved and the results are reported in Table 7. The second and the third columns of Table 7 show the first and the second inputs obtained by model (1), respectively. The fourth column of this table reports the optimal value of model (2.1) and the fifth column of Table 7 shows the sustainability radius of the cost efficiency of DMUs.

Then, we solve models (2.2) and (3.2) and report the results in Table 8. The second, the third and the fourth columns of this table show the first, the second and the third outputs obtained by model (2.2). The fifth column shows the optimal value of model (2.2) and the sixth column reports the sustainability radius of the revenue efficiency of units. Finally, we solve models (2.3) and (3.3) and report the results in Table 9. The second and the third columns of this table show the first and the second inputs and the fourth, the fifth and the sixth columns of Table 9 report the first, the second and the third outputs obtained by model (2.3). The seventh column shows the optimal value of model (2.3) and the eighth column reports the sustainability radius of the profit efficiency of units.

CONCLUSION

This study considered the sustainability radius of the cost efficiency, the revenue efficiency and the profit efficiency of units based on the sensitivity analysis. For this purpose, we eliminated the unit under evaluation and proposed some models to evaluate this unit. The most important feature of the proposed models is that these models can be applied to determine the sustainability region in which the efficiency of units do not change. Finally, we proposed some models for introducing the sustainability radiue of economic efficiency of units. The proposed approaches can help the managers to identify the permissible changes in the data of units such that their performances remain unchanged.

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DMU	x_1	x_2	y_1	y_2	y_3
1	2618	1106	2029864	680136	38714
2	1212	473	1003707	297719	18575
3	1721	531	1592960	408556	36658
4	2902	973	2596143	855467	75348
5	1389	447	1116161	337523	23803
6	1500	547	1476282	378658	22503
7	340	145	1300016	55003	5614
8	571	305	1052992	199780	26026
9	1168	369	1849711	326109	30967
10	921	372	1089975	209323	23847
11	920	316	33409	268723	15130
12	3236	1023	1954775	920215	56167
13	495	130	332741	136351	23423
14	1759	491	1465374	430407	35599
15	1357	390	1277752	368174	36006
16	2468	675	1825332	668467	32275
17	962	316	550700	247961	15618
18	745	272	1277899	217371	11671
19	1662	590	1916888	418205	21551
20	898	275	698945	209134	11748
21	1708	537	1702676	470437	32218

Table 6. the data of 21 medical centers in Taiwan

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DMU	x_1^*	x_2^*	z_1^*	θ^*
1	2368.1499	723.4161	157205.8730	0.7331
2	1005.2017	339.9891	69033.3138	0.5624
3	1320.6099	510.2261	95143.2705	0.5728
4	2966.8985	918.6140	197813.4153	0.1026
5	1145.5243	381.0418	78221.5145	0.5566
6	1223.9969	472.8665	88180.5150	0.6645
7	757.8939	276.7076	53474.7590	0.0000
8	750.7932	227.7426	49727.6748	0.0000
9	1024.4249	459.0865	78235.1742	0.1224
10	601.9081	316.8722	49266.9167	0.6917
11	975.5549	256.2063	61834.4136	0.4166
12	3340.6900	877.3529	211745.7564	0.5270
13	513.8912	274.4953	42339.7770	0.0000
14	1449.2918	494.4190	99827.4597	0.6608
15	1233.1191	427.8435	85439.4059	0.3253
16	2373.8187	676.6334	154186.1788	0.1180
17	900.1818	236.4114	57056.9807	0.2318
18	785.0480	249.7262	52807.9939	0.0000
19	1273.6896	580.2630	97934.4401	0.4699
20	707.7761	237.5887	48481.1314	0.3399
21	1556.5245	560.8614	109303.8957	0.2231

TABLE 7. The results of models (1) and (4)

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DMU	y_1^*	y_2^*	y_3^*	z_2^*	β^*
1	9915984.11	419540.1241	42821.2690	112107043.2276	0.7027
2	4240741.8483	179423.5793	18313.2552	47944513.0621	0.8165
3	4760748.2483	201424.7793	20558.8552	53823544.2621	0.3898
4	8723555.6414	369089.0966	37671.8759	98625816.5103	0.1010
5	4007635.5310	169560.9724	17306.6069	45309085.2828	0.5901
6	4904198.2897	207494.0759	21178.3310	55445345.9724	0.6900
7	583303.3802	107101.4398	12298.2976	9310445.2187	0.0000
8	2183262.1647	92372.6853	9428.2176	24683285.4059	0.0000
9	3308316.5793	139973.1517	14286.6621	37402801.9448	0.0000
10	3335213.4621	141111.1448	14402.8138	37706889.7655	0.8523
11	2833138.3172	119868.6069	12234.6483	32030583.7793	0.2194
12	9171837.0207	388055.6483	39607.7379	103693946.8552	0.5100
13	1165531.5862	49313.0345	5033.2414	13177138.8966	0.0000
14	4402123.1448	186251.5379	19010.1655	49769039.9862	0.5616
15	3496594.7586	147939.1034	15099.7241	39531416.6897	0.5144
16	6051798.6207	256048.4483	26134.1379	68419759.6552	0.6127
17	2833138.3172	119868.6069	12234.6483	32030583.7793	0.4649
18	2438650.7034	103178.0414	10531.0897	27570629.0759	0.0000
19	5289720.2759	223805.3103	22843.1724	59803938.0690	0.3814
20	2465547.5862	104316.0345	10647.2414	27874716.8966	0.1022
21	4814542.0138	203700.7655	20791.1586	54431719.9034	0.1212

TABLE 8. The results of models (2) and (5)

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DMU	x_1^*	x_2^*	y_1^*	y_2^*	y_3^*	z_3^*	ρ^*
1	2618.0000	1106.0000	6420614.3602	680136.0000	75670.4543	85787838.8418	0.1027
2	1212.0000	473.0000	2987909.2515	297719.0000	31043.3104	39061841.6220	0.1165
3	1582.0120	531.0000	3137617.8468	408556.0000	39120.6679	43551030.0819	0.0898
4	2902.0000	973.0000	2967537.1061	855467.0000	132764.2300	61603370.0417	0.0010
5	1321.3360	447.0000	2691444.4114	337523.0000	32358.3200	36976256.3098	0.0901
6	1500.0000	547.0000	3340778.0839	378658.0000	37968.5041	44901866.3567	0.0900
7	921.0000	372.0000	2522542.0237	209323.0000	23847.0000	31930471.2365	0.1523
8	920.0000	316.0000	1258447.1426	268723.0000	33477.3617	21576112.9970	0.0094
9	3236.0000	1023.0000	4090519.1496	920215.0000	116937.7887	71985992.8721	0.0100
10	1548.4406	491.0000	2488861.8226	430407.0000	40889.8607	37684016.8430	0.0616
11	1325.9356	390.0000	1605682.2293	368174.0000	46434.4961	28447733.7026	0.0144
12	2416.4574	675.0000	2314799.9178	668467.0000	98583.4960	47445935.0891	0.0127
13	949.3138	316.0000	1829375.3077	247961.0000	23713.4925	25680112.0106	0.0649
14	1662.0000	590.0000	3629316.5109	418205.0000	41159.5346	48890866.4654	0.0814
15	815.3189	275.0000	1644168.6271	209134.0000	20040.3760	22675246.3005	0.1022
16	1693.0313	537.0000	2724332.2910	470437.0000	44694.4023	41228907.1848	0.0212
17	2618.0000	1106.0000	6420614.3602	680136.0000	75670.4543	85787838.8418	0.1027
18	1212.0000	473.0000	2987909.2515	297719.0000	31043.3104	39061841.6220	0.1165
19	1582.0120	531.0000	3137617.8468	408556.0000	39120.6679	43551030.0819	0.0898
20	2902.0000	973.0000	2967537.1061	855467.0000	132764.2300	61603370.0417	0.0010
21	1321.3360	447.0000	2691444.4114	337523.0000	32358.3200	36976256.3098	0.0901

TABLE 9. The results of models (3) and (6)

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