Mathematical Analysis
& Convex Optimization

Vol. 4 (2023), No. 1, 155–162
https:\\ maco.lu.ac.ir
DOI: 10.22034/maco.4.1.12

Research Paper

MEASURING PRODUCTIVITY GROWTH IN THE MANUFACTURING INDUSTRY: A MALMQUIST GLOBAL INDEX APPROACH

ESMAEIL MOMBINI

Abstract. Data Envelopment Analysis (DEA) is a Nonparametric method for measuring of the performance of decision-making units (DMUs) - which do not need to have or compute a firm's production function, which is often difficult to calculate. In this article, we evaluate the units under review in terms of cost efficiency, and the units in terms of spending and production over several periods, and the rate of improvement or regression of each of these units. Considering the minimal use of resources and consuming less money, the improvement or retreat of the recipient's decision unit in terms of cost was examined by presenting a method based on solving linear programming models using the productivity index is Malmquist Global. Finally, by designing and solving a numerical example, we emphasize and test the applicability of the material presented in this article.

MSC(2010): 97M40; 97N60.
Keywords: Data envelopment analysis, econamic efficiency, Malmquist global index, improvements and regressions.

1. Introduction and Background

The concept of cost efficiency is that Measuring the ability of a DMU to produce outputs at the lowest input cost, Farrell began in 1957. He defined cost efficiency as the minimum cost to actual cost [13]. Since 1978, Charles, Cooper, and Rhodes have started data envelopment analysis which we are using now [10] and have become widely used by researchers to examine firms' performance with inputs, outputs, and technology. In 1978, Farrell introduced the DEA method by implementing CCR as a performance measurement method. This way it incorporates multi-input and multi-output production process features and provides unit-scale output when measuring efficiency. in 2014 ,Lee and Johnson proposed a short-term capacity planning approach [2]. Rostami at al (2016) investigated earnings inefficiency [1]. Kazutoshi Ando and his colleagues (2016) measured the minimum inefficiency gap in data envelopment analysis [9]. Rostami et al. start working on inefficiency in 2019 [15]. In 2015, Feng He and his colleagues examined the stability radius for uncertain boundaries . in 2015, Alahyar and Rostami reviewed the performance analysis for negative data and performance-based scalability. [11] In 2019, Peykani and Mohammadi proposed a domain-oriented size model of the feasibility of ranking warehouses in the presence of negative and non-linear data. In 1994, Fare and colleagues, Combined the ideas of Farrell efficiency measurement and productivity measurement with Q, and decomposed Malmquist's productivity index as a result of multiplying technical efficiency changes and technology boundary changes, from

Date: Received: January 15, 2023, Accepted: May 30, 2023 .

applications Important Malmquist Index Based on Data Envelopment Analysis (DEA) is the measurement of productivity changes between two specified time periods. Malmquist Index has some drawbacks despite being widely used, including when linear programming techniques. For the calculation of the index, the invariance occurs which is a fundamental problem in the data structure. And sufficient data were obtained to resolve the problem[5].

The MalmQuest Global Index can solve all three problems by specifying a fixed boundary. In 2012, Tohidi and colleagues used the Malmquist Global in cost estimation [17]. Emrooznejad and Yang used the malmquest Global to investigate carbon dioxide emissions [4]. Tohidi and Razavian used Malmquest Global in 2012 to calculate the profit of decision-making units. Not all articles present and research on the cost productivity of economic units were reviewed.

2. Preliminaries

A nonparametric method for measuring the performance of decision-making units (DMUs) is Data Envelopment Analysis (DEA). The first formulation was done by Charnes, Cooper, and Rhodes (CCR model) [3]. The original CCR model only used fixed-rate (CRS) technology, developed by Banker et al. If n is the decision unit (DMUj, j = 1, 2, 3,..., n) that uses m to produce the output S and the vectors $xj = (x_1j, ..., x_mj)$
and $Y_j = (y_1j, ..., y_sj)$ are the inputs and outputs, respectively. Therefore, $y_j \neq 0$, $x_j \neq 0$,$y_j \geq 0$,$x_j \geq 0$. Input-based technical performance for DMU0 is listed below

$$\begin{aligned} &\theta^* = \min \theta \\ &s.t: \sum_{J=1}^{n} \lambda_j X_j \leq \theta x_o \\ &\sum_{J=1}^{n} \lambda_j y_j \geq y_o \\ &\lambda_j \geq 0 \end{aligned} \tag{2.1}$$

Definition 2.1. cost efficiency
Consider the problem of minimizing the cost of a company with inputs to produce outputs and cost vector C.
We obtain the minimum total cost of producing the output vector y_o by solving the following model

$$\begin{aligned} &\min C^t x \\ &s.t: \sum_{J=1}^{n} \lambda_j X_j \leq x \\ &\sum_{J=1}^{n} \lambda_j y_j \geq y_o \\ &x \leq x_0 \\ &\lambda_j \geq 0, x \geq 0 \end{aligned} \tag{2.2}$$

Cost efficiency is the ratio of the minimum cost to the real cost.

$$CE_0 = \frac{C^t x^*}{C^t x_0}$$

$$0 \leq CE_i \leq 1$$

Definition 2.2. Calculating cost productivity improvement

Following the implementation of the CCR economic performance model, we seek to determine whether different decision makers have improved, regressed or unchanged in terms of their performance over different time periods. Accordingly, using the Malmquist Index, the performance improvement or regression of the units is analyzed. Malmquist index measurement requires calculation of distance functions. To solve these functions, we can use the linear programming method of comprehensive data analysis. In this regard, for each decision-making unit, according to the Malmquist Index between two time periods t and $t+1$, four distance functions must be calculated, which in turn requires solving four linear programming problems. Assuming constant-scale returns (assumed by Fier, Graskov, Norris, and Zhang in their analysis) [5]. Four issues will be addressed and solved for cost efficiency

$$\begin{aligned} &\min Cx \\ &s.t: \sum_{J=1}^{n} \lambda_j X_j^k \leq x \\ &\sum_{J=1}^{n} \lambda_j y_j^k \geq y_o^l \\ &x \leq x_0^l \\ &\lambda_j \geq 0, x \geq 0 \end{aligned} \tag{2.3}$$

(unit in time L – border in time K), $L = t,\ t+1,\ K = t,\ t+1$

Cost effectiveness:

$$CE_0 l^k = \frac{C^k x^*}{C^l x_0^l}$$

We can calculate the Malmquist Index for the cost efficiency of DMU0 using the following equation:

$$CMPI_O = \sqrt{\frac{CE_0(t+1)^t . CE_0(t+1)^(t+1)}{CE_0(t)^t . CE_0(t)^(t+1)}}$$

where if $CMPI_0 > 1$,Indicates the improvement in cost efficiency of DMU0 from time period t to time period $t+1$; if $CMPI_0 < 1$, Indicates the recovery of DMU0 cost efficiency from time period t to time period $t+1$; and if $CMPI_0 = 1$,Indicates the unchanged cost efficiency of DMU0 from time period t to time period $t+1$.

Definition 2.3. Malmquist Global

We have DMU0 generating units $(0 = 1, 2, ..., n)$ and defined time periods $t(t = 1, 2, ..., T)$. Global technology is defined as follows.

$$T_C^G = con[T_c^1, T_c^2, ..., T_c^T]$$

The Malmquist Productivity Index is defined as follows:

$$M_c^s(x^t, y^t, x^{(t+1)}, y^{(t+1)}) = \frac{D_c^s(x^{(t+1)}, y^{(t+1)})}{D_c^s(x^t, y^t)}$$

SO

$$D_c^s = \min\{\phi > 0 | (x, \frac{y}{\phi}) \in T_c^s\}, s = t,\ t+1$$

$$M_c = [M_c^t(x^t, y^t, x^{(t+1)}, y^{(t+1)}) M_c^{(t+1)}(x^t, y^t, x^{(t+1)}, y^{(t+1)})]$$

The global Malmquist Index defined onT_c^G is as follows:

$$M_c^G(x^t, y^t, x^{(t+1)}, y^{(t+1)}) = \frac{D_c^G(x^{(t+1)}, y^{(t+1)})}{D_c^G(x^t, y^t)} = EC_c.BPC_c.$$

3. MAIN RESULTS

Calculating cost productivity improvement and regression with the Malmquist Global Index

If the units are evaluated over time periods $t = 1, 2, 3, ..., T$ in the Malmquist Global Method, we examine all units in one place and the cost efficiency model for DMU0 at time k is given below. In this section, each DMU is compared to itself over different time periods, and it has progressed in two periods, such as p and q.

$$\begin{aligned} & \min C^k x \\ & s.t:\ \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j x_j^t \leq x \\ & \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j y_j^t \geq y_o^k \\ & x \leq x_0^k \\ & \lambda_j \geq 0, x \geq 0 \\ & k = 1, 2, ...T \end{aligned} \tag{3.1}$$

the cost efficiency comes from the following relationship:

$$CE_0^K = \frac{C^K x^*}{C^K x_0^K}$$

If we consider the productivity of DMU0 at time p compare to time q, we consider the solution of the model K = p, q and two problems arise and are solved. The MalmQuest Global Index is written as follows:

$$GCMPI_q^p = \frac{CE_0^p}{CE_0^q}$$

where if $GCMPI_q^p > 1$, the unit at time p have progressed to q; if $GCMPI_q^p < 1$, the unit at p has regressed to q; and if $GCMPI_q^p = 1$, the unit be unchanged at p relative to q.

Example 3.1. Numerical examples
For example, we consider 10 decision-making units with three inputs and two outputs over the three time periods t_3, t_2 and t_1 which are listed in Table 1. Table 2 shows the cost efficiencies for all units over the three time periods.

Table 6 show the progression and regression of units with the Malmquist Global Index. We find that the Global Malmquist has the characteristic of being out of the ordinary while the Malmquist lacks, this is the feature. For Unit 1 we are avoiding Malmquist Global with respect to cost efficiency index. From the ratio $GMPI_1^2$ and $GMPI_1^3$ we can get 1.129 for the $GMPI_2^3$. Also getting 0.883 from the ratio of $GMPI_2^3$ and $GMPI_1^3$ for getting $GRMPI_1^2$. Also, for unit 7 we need the ratio of $GRMPI_1^2$ and $GRMPI_1^3$ for getting the $GRMPI_2^3$ value which is 0.1074. Or from the ratio of $GRMPI_1^3$ and $GRMPI_2^3$ we can get the $GRMPI_1^2$ value which is 0.930.

	DMU_1			DMU_2			DMU_3			DMU_4			DMU_5			DMU_6			DMU_7			DMU_8			DMU_9			DMU_{10}		
	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
I_1	5	3	5	4	4	6	2	3	7	1	6	5	4	5	3	4	3	3	4	3	6	2	4	6	3	7	4	5	6	1
I_2	4	3	6	5	7	3	4	3	2	4	5	6	6	4	3	6	6	7	1	4	5	4	6	5	3	4	6	5	2	7
I_3	5	6	3	7	6	5	6	3	5	6	3	7	5	3	6	5	7	3	7	3	7	3	3	7	6	3	1	3	5	3
O_1	4	4	7	6	5	3	5	6	4	5	6	5	4	3	7	7	5	6	5	4	3	6	1	4	3	5	4	3	6	7
O_2	6	5	4	7	6	3	5	4	6	3	7	6	4	5	3	8	7	5	4	6	4	3	5	7	6	4	6	7	5	4

TABLE 1. the inputs and outputs data in three period for 10 DMUs.

DMU	Cost		
	t_1	t_2	t_3
1	0.750	0.618	0.673
2	0.760	0.632	0.335
3	0.765	1	0.579
4	0.660	0.912	0.571
5	0.531	0.608	0.693
6	1	0.708	0.772
7	0.593	1	0.523
8	1	0.678	0.275
9	0.828	0.570	1
10	0.928	0.647	1

TABLE 2. cost efficiency in three time for 10 DMUs.

DMU	CMPIO
1	0.98
2	0.84
3	0.08
4	0.31
5	1.11
6	0.28
7	0.76
8	4.88
9	8.86
10	1.10

TABLE 3. cost productivity improvement and regression with the malmquist index(t_1 to t_2).

DMU	CMPIO
1	0.11
2	0.62
3	0.67
4	0.31
5	0.23
6	0.31
7	2.44
8	0.20
9	4.97
10	0.37

Table 4. cost productivity improvement and regression with the malmquist index(t_1 to t_3).

DMU	CMPIO
1	32.48
2	2.92
3	0.89
4	8.10
5	0.28
6	3.38
7	0.41
8	0.20
9	1.33
10	1.86

Table 5. cost productivity improvement and regression with the malmquist index(t_2 to t_3).

DMU	CE_0^1	CE_0^2	CE_0^3	$GCMPI_1^2$	$GCMPI_2^3$	$GCMPI_1^3$
1	0.995	0.878	0.992	0.883	1.129	0.997
2	0.874	0.794	0.546	0.909	0.687	0.625
3	0.830	1	0635	1.203	0.635	0.765
4	0.841	0.896	0.797	1.100	0.889	0.947
5	0.650	0.887	0.722	1.364	0.814	1.111
6	1	0.867	0.865	0.857	0.998	0.865
7	0.779	1	0.917	1.282	0.917	1.176
8	0.972	0.889	0.939	0.914	1.055	0.965
9	0.769	0.743	0.892	0.967	1.200	1.161
10	1	0.904	0.687	0.904	0.759	0.687

Table 6. cost productivity improvement and regression with the malmquist global index.

4. ACKNOWLEDGEMENTS

The author is very thankful for the many helpful suggestions and corrections given by the refrees who reviewed this paper.

References

[1] Ahangari, T., Rostamy - Malkhalifeh, M. Measurement of profit inefficiency in presence of interval data using the directional distance function. Received 16 March 2016; accepted 26 May 2016.

[2] Allahyar, M. & Rostamy-Malkhalifeh, M. (2014). An improved approach for estimating returns to scale in DEA, Bulletin of the Malaysian Mathematical Sciences Society, (2) 37 (4) 1185 - 1194.

[3] Charnes A, Cooper ww and Rhodse, 1978. “Measuring the efficiency of decision making unit”, European Journal of Operation Research: 429 - 444.

[4] Emrouznejad, A. Yang, G-L. A framework for measuring global malmquist - Leunberger productivity index with CO2 emissions on Chinese manufacturing industries. Energy, 115, (2016), 840 - 856.

[5] Fare and Grosskopf S, Norris M, Zhang Z. 1994. productivity growth, technical progress and efficiency change in industrialized countries”, American economic Review, 84 (1): 66 - 83.

[6] F. Faraci, R. Livrea, *Infinitely many periodic solutions for a second-order nonautonomous system,* Nonlinear Anal. **54** (3): 417–429, 2003.

[7] Farrell, M.J., 1957. The measurement of productive efficiency. Journal of the Royal statistical society, series A 120: 253 - 281

[8] FengHe. Xiaoning Xu. Rong Chen. Na Zhang. Sensitivity and stability analysis in DEA with bounded uncertainty. Springer-Verlag. Berlin Hiedelberg 2015.

[9] Kazutoshi, A. Minamide, M. Kazuyuki, S. Shi, J. Monotonivity of minimum distance inefficiency measures for Data Envelopment Analysis. European Journal of Operational Research... (2016) 1 - 12.

[10] Lee, C.Y., & Johnson, A.L., (2014), Proactive data envelopment analysis: Effective production and capacity expansion in stochastic environments operational research, 232, 537 - 545.

[11] Maryam Allahyar, Mohsen Rostamy-malkhalifeh. Negative data in data envelopment Analysis: Efficiency analysis and estimating returns to sale computers & Industrial Engineering. (2015. 50360-8352 (15), 41–8.

[12] Mehdi toloo,Nazila Aghayi,Mohsen Rostamy-Malkhalifeh.Measuring Overall Profit Efficiency with interval data[J].Applied Mathematics and Computation,2008,201:640-649..

[13] M. Khodabakhshi, Y. Gholami, H. Kheirollahi, An additive model approach for estimating returns to scale in imprecise data envelopment Analysis, original Research Article Applied Mathematical Modelling. 34 (2010) 1247 - 1257..

[14] Pejman Peykani, Emran Mohammadi, Ali Emrouznejad, Mirsaman Pishvaee, Mohsen Rostamy-Malkhalifeh. Fuzzy Data Envelpment Analysis: An Adjustable Approach Expert Systems With Applications (2019) 50957-4174 (19) 30439 - 7.

[15] Pejman Peykani,Emran Mohammadi,Mohsen Rostamy-Malkhalifeh,Farhad Hosseinzadeh Lotfi.Fuzzy Data Envelopment Analysis Approach for ranking of stocks with an Application to Tehran stock Exchange.Advances in mathematical finonce & Applications ,4(1),(2019),31-43.

[16] R.Banker,A.Charnes,W.W.Cooper,SomeModels for estimating technical and scale inefficiencies in data

[17] Tohidi, G. Razavyan, S. Tohidnia, S. A global cost molmquist productivity index using data envelopment analysis, Journal of the operational Research Society, (2012). 63, 72 - 78.

(E. Mombini) Department of Mathematics, Ramhormoz Branch, Islamic Azad University,Ramhormoz, Iran.

Email address: `Eml_mombini@ yahoo.com`